The Impact of Measurement Error in Health in Health-Related Counterfactuals

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This Paper

- **Motivation**. Increasing body of research recognizes the importance of health in shaping economic decisions and outcomes.
  - Evidence suggests that "health" is complex and hard to measure.
  - Most studies assume that health is perfectly measured.
- **We ask**:

  How important is the imperfect observability of health to evaluate the costs of bad health?

- **Why should you care?**
  1. Better understanding of how health shapes decisions and outcomes.
  2. Gives a sense of how biased previous studies may be.
  3. Informative for future research.

This Paper

What We Do.

- Estimate canonical structural life-cycle model of savings and labor supply with health risk under two assumptions:
    1. Health is perfectly observable. (standard assumption)
    2. Health is not observable, but battery of noisy measures is. Health Model
- Counterfactuals in 2 estimated models (w/ and w/o ME in health).
    - We focus on the costs of bad health, as measured by labor earnings, hours worked, consumption, and assets.
    - Costs of bad health in outcome X: change the distribution of health shocks and look at $X(\text{counterfactual}) - X(\text{benchmark})$.
- By finding the difference in counterfactuals between the two models, we can quantify the bias introduced by ME in health.

This Paper

- **Findings**.
    1. Ignoring ME in health leads to underestimating the persistence of health and the time costs of being unhealthy.
    2. Lower persistence of health and lower time costs of bad health lead to underestimating the costs of bad health by 50–300%.

- **Contributions**.
    1. Estimate structural life-cycle model with health risk and ME in health taking into account ME in each stage of estimation.
    2. Speak to structural literature.

Details

Rest of the Talk

Structural Model

Data and Estimation

Main Results

Structural Model

- Individuals aged 50+ (ELSA core household members).
- Biannual life-cycle model: $t \in \{50\text{-}51, 52\text{-}53, \ldots, 86\text{-}87\}$.
- Individuals decide how much to work, consume and save.
  - *Partial equilibrium*.
- Health affects pecuniary resources, available time, health transitions.

  Details
- Government:
  - Taxes income. Tax system
  - Provides social security. Pension Benefits
  - Gives mean-tested transfers. Mean-tested programs
- At each t, the household's **state vector** is:

$$\boldsymbol{X}_t = \Big(\underbrace{H_t}_{\text{health}}, \underbrace{a_t}_{\text{assets}}, \underbrace{ae_t}_{\substack{\text{average}\\\text{earnings}}}, \underbrace{u_t}_{\substack{\text{persistent wage}\\\text{component}}} \Big).$$

Structural Model

Household head's decision problem:

$$\max \quad \mathbb{E}_0 \sum_{t=0}^{T} \beta^t \left[\frac{s_t(H_t)}{1-\gamma} \left\{ c_t^{\nu} \left[\overline{L} - \phi_P 1_{\{N_t>0\}} - N_t - \phi_H 1_{\{H_t=\text{Bad}\}} \right]^{1-\nu} \right\}^{1-\gamma} + \left[1 - s_t(H_t) \right] b(a_{t+1}) \right]$$

subject to

$$b(a_{t+1}) = \theta_B \frac{(\kappa_B + a_{t+1})^{(1-\gamma)\nu}}{1-\gamma}$$

Budget constraint

Borrowing constraint

Transition functions

Initial conditions

Borrowing and budget constraints Numerical Procedure

Data

- **We use data from ELSA** (*English Longitudinal Study of Ageing*):
  - Survey data
  - Representative of the old English population.
  - Individuals aged +50 (and their partners).
  - Study started in 2002; today, 9 waves (bi-annual interviews).
- **Why ELSA, why the UK?** Avoid unnecessary complications.
  - No need to model employer-provided health insurance and medical expenditures. (Important factors in the environment of the US)
  - NHS provides universal health care (to UK ordinarily residents).
  - Private health care used by approx. 10%, as a top-up to NHS.

Estimation

Two-step estimation procedure.

1. Estimate some parameters outside the model and set some others to values taken from the literature.

   Taxes & Other Parameters | Wages | Pensions | Spousal earnings | Average earnings | Initial distribution

   ⋆ Health process and ME model. (next slide)

2. Estimate remaining parameters inside model using Indirect Inference.

Parameters Estimated Outside the Model

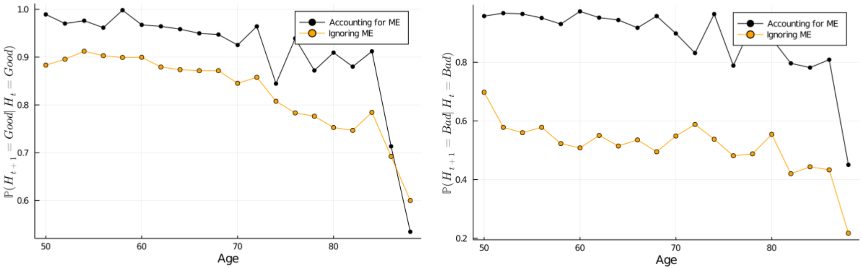
Process for health and ME model.

1. Ignoring ME: health and its dynamics identified and estimated using SRHS and empirical transition probabilities between health states.

2. Acknowledging ME: true health unobservable; only noisy measures. Use non-stationary hidden Markov model for health.

ME Model Identification Estimation Algorithm

Figure: **Higher persistence of health when taking into account ME**.



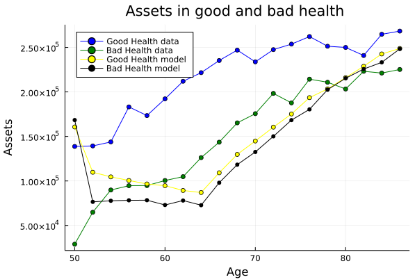
Parameters Estimated Inside the Model

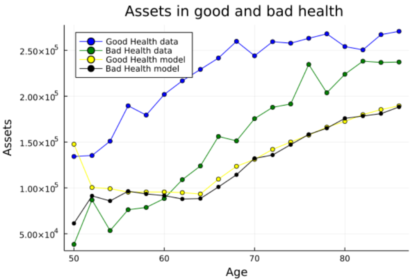
We target profiles for assets, hours worked, participation (all by measured health status), and the coefficients of the FE regression for wages

Parameter	**Acknowledging ME**	**Ignoring ME**
β: bi-annual discount factor	0.76	0.80
γ: CRRA coefficient	3.93	3.71
ν: consumption weight in utility function	0.48	0.46
ϕ_P: fixed cost of participation	1,098	1,076
θ_B: weight on bequest	0.066	0.076
$\boldsymbol{\phi_H}$: **time cost of bad health**	1,851	875
C^y_{min}: consumption floor when young	6,927	4,573
C^o_{min}: consumption floor when old	13,776	12,409
a_0: constant term of wage profile	2.21	1.92
a_1: linear age-term of wage profile	0.005	0.012
a_2: quadratic age-term of wage profile	-0.005	-0.005
a_H: health coefficient of wage profile	0.0224	0.0256

Table Notes. Parameters ϕ and C_{min} should be interpreted in terms of bi-annual hours and bi-annual GBP, respectively.

Model Fit: accounting for ME (left), ignoring it (right)





- Similar fit when taking into account and ignoring ME.
- Missing the levels, roughly capturing the trends.
- Some trouble in fitting data due to the initial distribution of states.

Data Profiles | Model Fit: Hours & Assets | Model Fit: Wage profiles | Initial distribution of states

Results: The Costs of Bad Health

- We follow De Nardi, Pashchenko, and Porapakkarm (2018) and calculate the costs of bad health as measured by many outcomes.

- The **exercise** consists of:

  1. Simulating the model imposing everyone is always in good health.

     $\rightarrow$ Individuals' histories of earnings, hours worked, consumption, assets.

  2. Simulating the model letting health evolve according to estimated transition matrix.

     $\rightarrow$ Individuals' histories of earnings, hours worked, consumption, assets.

  3. Find differences in annual means between histories in (1) and (2).

- We do this exercise twice (**w/ and w/o taking into account ME**), and then find difference in counterfactuals between models.

The Costs of Bad Health (**all individuals**)

- Ignoring ME in health leads to substantially underestimating the costs of bad health for all outcomes (especially for earnings).
- Mainly **two forces driving the results**:
  1. Higher persistence of health when taking into account ME.
  2. Higher time costs of bad health when taking into account ME.

Outcome	**Acknowledging ME**	**Ignoring ME**	**Difference** (%)
Earnings	972	456	113%
Hours worked	106	62	71%
Consumption	1,773	1,081	64%
Assets	21,827	16,632	31%

Notes. All variables are means expressed in annual terms. Mean earnings and hours worked are computed up to age 64 (inclusive). Units of earnings, consumption, and assets are GBP.

The Costs of Bad Health (**initially unhealthy**)

- Costs of bad health higher than for the overall population. (obvious since health is persistent)
- Persistence of health main force behind differences in columns.
  - If health was iid, the expected time that an initially healthy and an initially unhealthy would spend in bad health would be more similar than if health was persistent. (think of differences between tables).

Outcome	**Taking into account ME**	**Ignoring ME**	**Difference** (%)
Earnings	3,962	987	302%
Hours worked	433	127	240%
Consumption	3,822	1,618	136%
Assets	32,047	21,039	52%

Notes. All variables are means expressed in annual terms. Mean earnings and hours worked are computed up to age 64 (inclusive). Units of earnings, consumption, and assets are GBP.

Taking Stock

- **Question**: How important is the imperfect observability of health to evaluate the costs of bad of health?
- **Method**:
  - Estimate a canonical structural life-cycle model of savings and labor supply with health risk under two assumptions:
    1. Health is perfectly observable.
    2. Health is not observable, but noise measures are.
  - Look at costs of bad health (as measured by earnings, hours worked, consumption and assets) to assess bias introduced by ME in health.
- **Findings**.
  1. Ignoring ME in health leads to underestimating the persistence of health and the time costs of bad health.
  2. This leads to underestimating the costs of bad health by 50–300%.
- Our results suggests that previous studies likely to substantially underestimate the lifetime costs of bad health.

Thank You!

Extra Slides

Measurement Error Model for Health

1. At each point in time, an individual can be in one of $r-1$ different unobserved health states $H_t \in \{1, 2, \ldots, r\}$, where $r =$ dead.
   - E.g., $H_t \in \{\text{Good health } (=1), \text{ Bad health } (=2), \text{ Dead } (=3)\}$.
2. Health evolves according to a non-stationary Markov model with transition matrices $\{K_t\}$, where:
$$K_t(j,k) := \mathbb{P}_t(H_{t+1} = k | H_t = j).$$
3. The econometrician cannot observe true health status (except for mortality), but can observe at least 3 discrete noisy measures:
$$Y_t^m \in \{1, \ldots, \kappa_m, \kappa_{m+1}\}.$$
   - $\boldsymbol{Y}_t = \left\{\text{Pain severity, \#ADL + \#IADL limitations, mobility cond.}\right\}$.
4. The conditional distribution of Y_t^m is given by the matrix P_t^m, where
$$P_t^m(c,j) := \mathbb{P}(Y_t^m = c \mid H_t = j)$$

Back to Intro Back to Estimation

Our Contributions Back

1. **Estimate structural life-cycle model with health risk and ME in health, taking into account ME in each stage of estimation.**
   - Most papers assume perfect observability of health.
   - Papers that do not, *only partially* address ME in health.
     - Ignore ME when estimating initial distribution of states, spousal income, preference parameters (French, 2005; Amengual et al., 2021).
     - Impose restrictive parametric assumptions on health dynamics.
     - Restrictive identification (French, 2005) or identification not discussed (Amengual et al., 2021).
     - We guarantee identification of the dynamics of health and its measurement system under less restrictive assumptions.
2. **Speak to structural literature**.
   - Previous studies (Capatina, 2015; De Nardi et al., 2018) likely to highly underestimate lifetime costs of bad health.
   - Future research needs to worry about ME in health.

Details: Wages and Spousal Earnings

- **Log-wages** are:

$$\log W_t(H_t, t) = a_0 + a_1 t + a_2 t^2 + a_H 1_{\{H_t = \text{Good}\}} + u_t,$$

where

$$u_t = \rho u_{t-1} + \xi_t, \qquad \xi_t \sim i.i.d.$$

- **Spousal income** is a deterministic function of health and age:

$$ys_t = \begin{cases} ys(t, H_t), & \text{if } t \leqslant R_a + 1 \\ 0, & \text{if } t > R_a + 1 \end{cases}.$$

(motivated by the data).

Back

Details: Tax System

$$\text{Income taxes}(ti, t) = \begin{cases} 0, & \text{if} \quad ti \leqslant \kappa_1^t \\ 0.2(ti - \kappa_1^t), & \text{if} \quad \kappa_1^t < ti \leqslant \kappa_2^t \\ 0.2(\kappa_2^t - \kappa_1^t) + 0.4(ti - \kappa_2^t), & \text{if} \quad \kappa_2^t < ti \end{cases},$$

Table: Income Tax Thresholds from O'Dea (2018)

	Age		
Parameter	< 64	64–73	$\geqslant 74$
κ_1	16,210	21,000	21,200
κ_2	84,940	89,740	89,940

Back

Details: Pension Benefits

- **Public pension benefits** are a function of average earnings at 64:

$$\mathrm{pbb}_t = \begin{cases} g(ae_{64}), & t \geqslant R_a \\ 0, & \text{otherwise} \end{cases}.$$

- **Private benefits** also a function of average earnings at age 64:

$$\mathrm{privben}_t = \begin{cases} f(ae_{64}), & t \geqslant R_a \\ 0, & \text{otherwise} \end{cases}.$$

Back

Details: Mean-tested programs

- The government gives transfers to household heads in order to ensure a minimum level of consumption $C_{\min,t}$.
  - $C_{\min,t}$ is allowed to depend on age.
  - This intends to capture the fact that, in the UK, retirees face different mean-tested programs than non-retirees.

- **Government transfers** are parametrized as:

$$\text{tr}_t = \begin{cases} \max\left\{0, C_{\min,t} - \left[W_t N_t + (1+r)a_t + ys(t, H_t)\right]\right\}, & t < R_a \\ \max\left\{0, C_{\min,t} \right. & \\ \qquad \left. - \left[W_t N_t + (1+r_t)a_t + ys(t, H_t) + \text{pbb}_t + \text{privben}_t\right]\right\}, & t \geqslant R_a \end{cases}.$$

$C_{\min,t}$ changes at $t = R_a$:

$$C_{\min,t} = \begin{cases} C^y_{\min}, & t < R_a \\ C^o_{\min}, & t \geqslant R_a \end{cases}.$$

Back

Borrowing and Budget Constraints

- Households face a **no-borrowing constraint**:

$$a_{t+1} \geqslant 0, \qquad \forall t.$$

- Save at **constant interest rate** r.

- Hence, the household's **budget constraint** is:

$$c_t + a_{t+1} = y_t(ra_t + W_t N_t) + ys(t, H) + a_t + tr_t \text{ if } t < R_a,$$

$$c_t + a_{t+1} = y_t(ra_t + W_t N_t + \text{privben}_t + \text{pbb}_t) + ys(t, H) + a_t + tr_t \text{ if } t \geqslant R_a.$$

Back

Model Solution: Numerical Procedure

- There are four states (apart from age): Assets, health, average earnings, and the stochastic component of wages.
  - Health already discrete. Rest are discretized and placed on a grid.
- There two continuous choices: Assets tomorrow and hours worked.
  - Also discretized and placed on a grid.
- Wage shock discretized using extension to life-cycle models of the Rouwenhorst method by Fella,Gallipolli and Fan (2019).
  - This produces a transition matrix and a grid for each age.
- Value function at each age found by backward induction.
  - Given value function at $t+1$ problem at t solved by grid search.
- Expectations of the value function are taken using the transition function for health and for the discretized wage shocks.
- Average earnings tomorrow can be outside the grid $\Longrightarrow$ use linear interpolation.

Back

Parameters from the Literature

Back

Parameter	Value	Source
κ_B: curvature of bequests	650,000	O'Dea (2018)
$\overline{L}$: total endowment of bi-annual hours	8,760	12 daily hours
r: interest rate non-housing wealth	0.0323	O'Dea (2018)

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	Age		
Parameter	< 64	64–73	$\geqslant 74$
κ_1	16,210	21,000	21,200
κ_2	84,940	89,740	89,940

$$\text{Income taxes}(ti, t) = \begin{cases} 0, & \text{if } \; ti \leqslant \kappa_1^t \\ 0.2(ti - \kappa_1^t), & \text{if } \; \kappa_1^t < ti \leqslant \kappa_2^t \\ 0.2(\kappa_2^t - \kappa_1^t) + 0.4(ti - \kappa_2^t), & \text{if } \; \kappa_2^t < ti \end{cases},$$

Parameters Estimated Outside the Model Back

- **Wage parameters**. FE regression:

$$\log(W_{it}^{\text{data}}) = a_0 + a_1 t + a_2 t^2 + a_H \mathbf{1}_{\{H_{it}=\text{Good}\}} + \underbrace{\eta_i + u_{it} + m_{it}}_{=\varepsilon_{it}}$$

$$u_{it} = \rho u_{it-1} + \xi_t, \quad \xi_1 \sim \mathcal{N}(0, \sigma^2_{\xi,1}), \quad \xi_t \sim \mathcal{N}(0, \sigma^2_{\xi,t}) \ \forall t > 1,$$

$$m_{it} \sim \mathcal{N}(0, \sigma^2_m), \qquad \eta_i \sim \mathcal{N}(0, \sigma^2_\eta), \qquad \eta_i \perp \xi_t, \ \forall i, t.$$

Wage parameters biased due to selection of workers in labor market.

Identification of wage-shock parameters

Minimum Distance Estimates **Wage-shock parameter**	**Health (with DLs)**	**Health (with SRHS)**
ρ: autocorrelation	0.8764	0.8790
σ^2_η: variance fixed effect	0.0806	0.0775
$\sigma^2_{\xi,1}$: variance innovations at $t=1$	0.1949	0.1991
$\sigma^2_{\xi,t}$: variance innovations at $t>1$	0.0581	0.0577
σ^2_m: variance measurement error	0.1645	0.1645

Parameters Estimated Outside the Model Back

- **Pension parameters**. Estimate parameters that relate public and private pension benefits with avg. earnnigs with OLS:

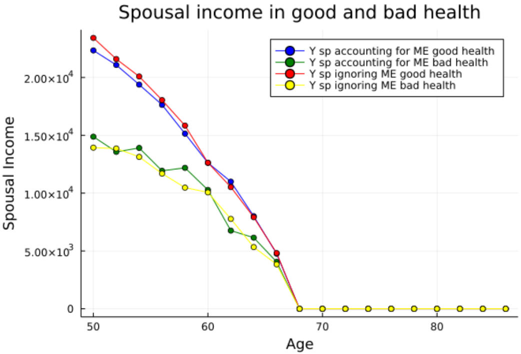
$$pbb_{i,65} = ss_1 ae_{i,64} + ss_2 ae_{i,64}^2 + \varepsilon_i, \qquad \text{for} \quad ae_{i,64} \leqslant \widehat{ae}_{ss},$$
$$privben_{i,65} = pp_0 + pp_1 ae_{i,64} + pp_2 ae_{i,64}^2 + \xi_i,$$

($\widehat{ae}_{ss}(=75k)$: threshold at which quadratic relationship between *pbb* and *ae* starts to decrease.)

Parameter	Value	S.E.
ss_1	0.6518	0.0006
ss_2	−3.56E-06	1.31E-08
pp_0	5,980.80	591.35
pp_1	0.3426	0.0266
pp_2	5.23E-07	2.45E-07

Parameters Estimated Outside the Model

- **Spousal earnings**. Empirical counterpart of the spousal income function is given by $\text{ys}(t, H) = \mathbb{E}\left[\text{ys}_{it} \mid H_{it}\right]$. Identification & Estimation with ME



Back

Average Earnings

Since ELSA starts at age 50, we need retrospective information on employment and earnings to construct measure of average earnings

- **Ideally**. Administrative data to obtain average earnings.
  - Restricted to UK-affiliated researchers.
- **Go-around #1**. ELSA Life History + ELSA surveys to construct employment spells and earnings histories since job market entry.
  - Not very effective in practice.
  - Imputed average earnings do not exhibit the relationship with pension benefits documented by many others.

    → Very noisy imputation procedure.
- **Go-around #2**. ELSA data + data simulated by O'Dea (2018).
  - O'Dea (2018) provides a good fit of earnings profiles (admin data).
  - Implicit assumption: our cohort (1950–57) is similar to his (1935–50).

Imputation of Average Earnings with O'Dea Data

1. Obtain parameters $\{\hat{\beta}_0^j, \hat{\beta}_1^j, \hat{\beta}_2^j\}_{j=1}^4$, where j indexes household type, from OLS regression in data simulated by O'Dea (2018):

$$ae_{i,64}^j = \beta_0^j\big(1-1\{\text{pbb}_{i,65}^j \geqslant 29.13k\}\big)\text{pbb}_{i,65}^j + \beta_1^j 1\{\text{pbb}_{i,65}^j \geqslant 29.13k\} + \beta_2^j\big(1-1\{\text{pbb}_{i,65}^j \geqslant 29.13k\}\big)\text{privben}_{i,65}^j + \varepsilon_i^j. \quad (1)$$

(Reference group is those receiving more than 29.13k GBP in *pbb*; for others, avg. earnings are linear in *pbb* and *privben* at 65)

2. Use $\{\hat{\beta}_0^j, \hat{\beta}_1^j, \hat{\beta}_2^j\}_{j=1}^4$ and similar household classification to generate $ae_{i,64}$ for individuals in our sample according to (1) without ε_i^j term.

3. Recover average earnings at age 50 from:

$$ae_{i,64}^j = \frac{\text{Emp years}_{i,50}^j \cdot ae_{i,50}^j + \text{Earnings ELSA}_i^j}{\text{Emp years}_{i,50}^j + \text{ELSA empl years}_i^j}.$$

Back

Initial Distribution of States

1. Ignoring ME: Obtain initial distribution of states directly from data (assets, average earnings, health), simulating initial wage-shocks according to estimated initial distribution of wage shocks.

2. Acknowledging ME:

   - Use previously-estimated initial probability distribution of true health.

   - Estimate joint distribution of assets & avg. earnings given true health

     - Assume $(\log a_0, \log ae_0)$ are jointly log-normal given true health

$$\begin{pmatrix} \log a_0 \\ \log ae_0 \end{pmatrix} \sim \mathcal{N}(\mu_H, \Sigma_H).$$

   - Use previously-estimated initial distribution of wage shocks. (assuming these are independent of the rest of states)

Back to Estimation Back to Model Fit

Identification of ME Model for Health

Mild extension of Garcia-Vazquez (2021) to show that model is identified.

Assumption

A1. *Access to three conditionally-independent noisy measures Y_t^m of the unobserved state H_t.*

A2. *(i)* $\mathbb{P}(H_{t+1} \mid H_t, Y_t^1) = \mathbb{P}(H_{t+1} \mid H_t)$ *for* $t = 0, \dots, T-1$.
(ii) $\mathbb{P}(Y_t^1 \mid H_{t+1}, Y_{t+1}^1) = \mathbb{P}(Y_t^1 \mid H_{t+1})$ *for* $t = 0, \dots, T$.

A3. P_t^m*, the conditional distribution of* Y_t^m*, is full rank for* $m \in \{1, 2, 3\}$.

A4. *The cross-sectional distribution of the underlying state,* π_t*, is such that* $\pi_t(c) > 0$ *for each* $c \in \{1, \dots, r\}$.

A5. $\exists i, m^*$ *known by the researcher s.t. for row* i *of matrix* P^{m^*} *we have* $P^{m^*}(i,j) \neq P^{m^*}(i,j')$ *for all columns and* $P^{m^*}(i,j)$ *is monotone in* j.

Theorem (Identification)

Suppose Assumptions A1–A5 hold. Then the model is identified.

Identification of ME Model for Health

- **Identification idea**:
  1. *Cross-sectional step* identifies cross-sectional parameters $\pi_t, \{P_t^m\}$.
  2. *Longitudinal step* identifies transition matrices for state, $\{K_t\}_{t=0}^{T-1}$.

- **Proof**. Identification argument for $K_0, \pi_0, \pi_1, \{P^m\}_{m=1,2,3}$.
  - From Bonhomme et al. (2016), Theorems 2–3: $\pi_0, \pi_1, \{P^m\}_{m=1,2,3}$ are identified. WTS: K_0 is identified.
  - Note that the joint distribution of noisy measure $m = 1$ at $t = 0, 1$ is:

$$\mathbb{P}(Y_0^1, Y_1^1) = P^1 \Pi_0 K_0 \Pi_1^{-1} (P^1 \Pi_0)',$$

    where $\mathbb{P}(Y_0^1, Y_1^1)(i, j) = \mathbb{P}(Y_0^1 = i, Y_1^1 = j)$.
  - Let $\Omega_0 = P_1 \Pi_0$ and $\Omega_1 = \Pi_1^{-1}(P^1 \Pi_1)'$. Since P^1 is full rank (A3):

$$K_0 = (\Omega_0' \Omega_0)^{-1} \Omega_0' \mathbb{P}(Y_0^1, Y_1^1) \Omega_1' (\Omega_1' \Omega_1)^{-1}.$$

  - Since $\mathbb{P}(Y_0^1, Y_1^1)$ is observable and Π_0, Π_1, P^1 are identified, this completes the proof.

Back

ME Model: Estimation Algorithm

- **First Step of Constrained Baum–Welch**:
  - At each $t = 1, \ldots, T$ restrict the sample to observations that are not missing or death, i.e., $Y_t^1 \neq \kappa_1, -7$.
  - Use ML and the EM algorithm to get $\sqrt{N}$-consistent and asymp. normal estimates of P^1 and $\tilde{\pi}_t = \{\mathbb{P}(H_t = s \mid H_t \neq r\}_{s=1,\ldots,r}$.
  - For each t, calculate proportion of people that dies between t and $t+1$ as:

$$\text{Prop. of deaths}_{t,t+1} = \hat{\mathbb{P}}(Y_{t+1}^1 = \kappa_1 \mid Y_t^1 \neq \kappa_1, -7).$$

  - Let

$$\pi_{t+1}^{H_t \neq r} = \left\{\mathbb{P}(H_{t+1} = s \mid H_t \neq r)\right\}_{s=1,\ldots,r}.$$

  - A consistent estimate of this object is:

$$\hat{\pi}_{t+1}^{H_t \neq r} = \left(\hat{\tilde{\pi}}_{t+1}(1 - \text{Prop. of deaths}_{t,t+1}),\ \text{Prop. of deaths}_{t,t+1}\right),$$

  where $\hat{\tilde{\pi}}_{t+1}$ denotes the estimate for $\tilde{\pi}_{t+1}$.

ME Model: Estimation Algorithm

- **Second Step of Constrained Baum–Welch**:
  - For each $t = 1, \dots, T-1$, restrict the sample to observations to those that are non-missing in t and $t+1$ and non-death in t.
  - Estimate K_t iterating between an E and a M step until convergence.
    - **E step**: Let Q_1 be the emission matrix for Y^1 expanded to include mortality. Given estimates for $\hat{\hat{\pi}}_t, \hat{\pi}_{t+1}^{H_t \neq r}, Q^1, \{Y_{i,\tau}^m\}_{\tau=t,t+1}$ and a guess for $K_t^{(h)}$ calculate the filtered probabilities:

$$\hat{v}_{i,k,j} := \mathbb{P}\big(H_{i,t+1} = j, H_{i,t} = k \mid Y_{i,t}^1, Y_{i,t+1}^1, \{\hat{\pi}_\tau\}_{\tau=t,t+1}, \hat{Q}^1, K_t^{(h)}\big).$$

These filtered probabilities can be computed as:

$$\hat{v}_{i,k,j} = \frac{\hat{Q}^1(y_{i,t}^1, k)\hat{\pi}_t(k)K^{(h)}(k,j)\hat{Q}^1(y_{i,t+1}^1, j)}{\sum_{j=1}^r \sum_{k=1}^r \hat{Q}^1(y_{i,t}^1, k)\hat{\pi}_t(k)K^{(h)}(k,j)\hat{Q}^1(y_{i,t+1}^1, j)}.$$

ME Model: Estimation Algorithm

- **M step**: Calculate the new guess $K_t^{(h+1)}$ as:

$$K_t^{(h+1)} = \arg\max_{K} \quad \sum_{i=1}^{N} \left\{ \sum_{k=1}^{r} \sum_{j=1}^{r} v_{i,j,k} \log\left(K(k,j)\right) \right\}$$

$$\text{s.t.} \quad \sum_{j=1}^{r} K_t(k,j) = 1, \quad \forall k,$$

$$\sum_{j=1}^{r} K_t(j,c) \hat{\hat{\pi}}_t(j) = \hat{\pi}_{t+1}^{H_t \neq r}(c)$$

Back

Identification of Wage-shock Parameters

- Let
$$\varepsilon_{it} = \eta_i + u_{it} + m_{it}, \qquad t = 1, \ldots, T.$$
denote the residuals from the wage equation (in levels).

- Using recursion on u_{it} $(= \rho u_{it-1} + \xi_t)$, we can write:
$$\varepsilon_{it} = \eta_i + m_{it} + \sum_{\tau=1}^{t} \rho^{t-\tau} \xi_\tau, \qquad t = 1, \ldots, T.$$

- Identification:
$$1 + \rho = \frac{\mathrm{Cov}(\varepsilon_{i4} - \varepsilon_{i2}, \varepsilon_{i1})}{\mathrm{Cov}(\varepsilon_{i3} - \varepsilon_{i2}, \varepsilon_{i1})}, \qquad \sigma^2_{\xi,1} = \frac{\mathrm{Cov}(\varepsilon_{i4} - \varepsilon_{i2}, \varepsilon_{i1})}{\rho(\rho^2 - 1)},$$
$$\sigma^2_m = (\rho - 1)\sigma^2_{\xi,1} - \mathrm{Cov}(\varepsilon_{i2} - \varepsilon_{i1}, \varepsilon_{i1}), \qquad \sigma^2_{\xi,t} = \mathrm{Var}(\varepsilon_{i2} - \varepsilon_{i1}) - 2\sigma^2_m - (\rho - 1)^2 \sigma^2_{\xi,1},$$
$$\sigma^2_\eta = \mathrm{Var}(\varepsilon_{i2}) - \rho^2 \sigma^2_{\xi,1} - \sigma^2_m - \sigma^2_{\xi,t}.$$

- Target (+80) additional statistics of the wage-shock process, to increase precision of MD estimation.

Back

Spousal Earnings: Identification & Estimation with ME

- The empirical counterpart of the spousal income function is given by:

$$\text{ys}(t, H) = \mathbb{E}\big[\text{ys}_{it} \mid H_{it}\big].$$

- Health is not directly observable, but this can be estimated using Minimum Distance. Identifying assumption:

Assumption (Exclusion restriction)

$$\mathbb{E}\big[ys_t \mid Y_t^1, H_t = c\big] = \mathbb{E}\big[ys_t \mid H_t = c\big].$$

This amounts to saying: "*Given true health, suffering from problems, say, with mobility conditions does not predict spousal income*"

Spousal Earnings: Identification & Estimation with ME

- Under the exclusion restriction, we can write the expectation of spousal earnings given Y_t^1 as:

$$\mathbb{E}\big[\mathrm{ys}_t \mid Y_t^1 = y\big] = \sum_{c=1}^{2} \mathbb{E}\big[\mathrm{ys}_t \mid H_t = c\big]\mathbb{P}\big(H_t = c \mid Y_t^1 = y\big), \qquad y = 1, \ldots, \kappa_1.$$

- This can be written as the following linear system:

$$\mathbb{P}(Y_t^1)^{-1} P^1 \Pi_t \mathbb{E}\big[\mathrm{ys}_t \mid H_t\big] = \mathbb{E}\big[\mathrm{ys}_t \mid Y_t^1\big],$$

  where:
  - $\mathbb{E}\big[\mathrm{ys}_t \mid H_t\big]$: column vector whose i-th element is $\mathbb{E}\big[\mathrm{ys}_t \mid H_t = i\big]$.
  - $\mathbb{E}\big[\mathrm{ys}_t \mid Y_t^1\big]$: column vector whose i-th element is $\mathbb{E}\big[\mathrm{ys}_t \mid Y_t^1 = i\big]$.
  - $\mathbb{P}(Y_t^1)$: diagonal matrix with cross-sectional distribution of Y_t^1 at each t in the diagonal.

- This system has at most one solution if P^1 is full rank and if π_t has non-zero elements for all t. (already required for ME identification)

- Estimation by imposing this linear-system of restrictions in a finite sample using Minimum Distance.

Back

Data Profiles

- We pool individuals from different birth cohorts and assume that differences in profiles across cohorts driven solely by cohort effects.
  - This responds to data limitations.
- Classify individuals in:
  - 19 two-year age groups (from 50–51 to 86–87).
  - 4 cohort groups (born before 1935, born between 1935–1943, born between 1943–1950, born after 1950).
  - 2 health groups (healthy and unhealthy).
- Run regression

$$y_{i,w} = \gamma_0^{y,m} + \eta_a^{y,m} + \eta_c^{y,m} + \gamma_a^{y,m}\left(a_{iw} \times 1_{\{\text{Health}_i^m = \text{Good}\}}\right) + u_{i,w}^{y,m},$$

y: targeted variable i: individual w: wave
m: health indicator a: age c: cohort

to obtain estimates $\left\{\hat{\gamma}_0^{y,m}, \{\hat{\eta}_a^{y,m}, \hat{\gamma}_a^{y,m}\}_{a=1}^{19}, \{\hat{\eta}_c^{y,m}\}_{c=1}^{4}\right\}$ for each y.

Data Profiles

- Use estimates $\left\{\hat{\gamma}_0^{y,m}, \{\hat{\eta}_a^{y,m}, \hat{\gamma}_a^{y,m}\}_{a=1}^{19}, \{\hat{\eta}_c^{y,m}\}_{c=1}^{4}\right\}$ for each (y,m)-pair to generate y profiles by health status according to:

$$
\begin{aligned}
y_a^{\text{good health}(m)} &= \hat{\gamma}_0^{y,m} + \hat{\eta}_a^{y,m} + \hat{\eta}_{c=4}^{y,m} + \hat{\gamma}_a^{y,m} a, \qquad a \in \{1,\dots,19\}, \\
y_a^{\text{bad health}(m)} &= \hat{\gamma}_0^{y,m} + \hat{\eta}_a^{y,m} + \hat{\eta}_{c=4}^{y,m}, \qquad a \in \{1,\dots,19\}.
\end{aligned}
$$

y: targeted variable a: age m: health indicator c: cohort

$c = 4$ (our cohort; those born in 1950–57)

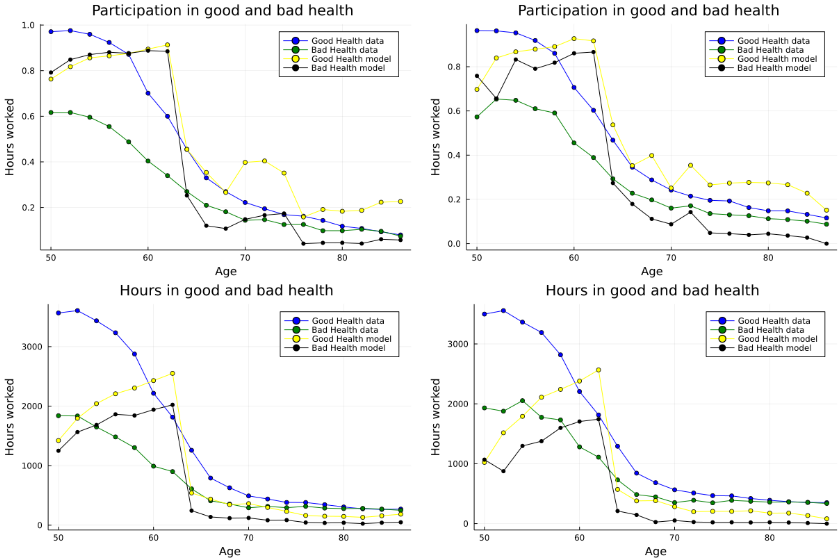
- The vectors

$$
\begin{aligned}
\mathrm{y}^{\text{good health}(m)} &= \left(y_1^{\text{good health}(m)}, \dots, y_{19}^{\text{good health}(m)}\right), \\
\mathrm{y}^{\text{bad health}(m)} &= \left(y_1^{\text{bad health}(m)}, \dots, y_{19}^{\text{bad health}(m)}\right),
\end{aligned}
$$

give the y profiles for our cohort where $m \in \{\mathrm{SRHS}, \mathrm{DLs}\}$.

Back

Model Fit: accounting for ME (left), ignoring it (right)



Back

Model Fit: Estimated Wage Profiles

- Similar fit for a_0 with and without ME (± 0.16).
- Better fit for (a_1, a_2, a_H) in model that ignores ME.

$$\log(W_{it}^{\text{data}}) = a_0 + a_1 t + a_2 t^2 + a_H \mathbf{1}_{\{H_{it}=\text{Good}\}} + \eta_i + u_{it} + m_{it}$$

	Accounting for ME		Ignoring ME	
Parameter	**Data**	**Model**	**Data**	**Model**
a_0	2.09	2.24	2.1	1.94
a_1	0.058	0.003	0.06	0.013
a_2	-0.02	-0.004	-0.002	-0.005
a_H	0.024	0.002	0.009	0.013

Back