On the Efficiency of Competitive Equilibria with Pandemics

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Modeling Pandemics

- **Typical economic approach**:
  - Treats economic effects of pandemics in exactly the same way as those of climate change—as *global externalities*.
- **Epidemiological approach**:
  - Transmission occurs in meetings.
  - But have little to say about meetings and economic outcomes.
- **Our approach**:
  - Model relationship between meetings and economic activity.
  - Recognize individuals have some control over meetings.
  - Implies pandemics create *local externalities*.

Typical economic approach vs. Our approach

Our Framework

- Embed SIR framework in search/matching/wage-posting model.
    - Types of people who meet each other endogenously determined.
- Adopt metaphor of islands from search literature.
    - Islands characterized by wage menus depending on infection status.
        - Allows firms to discriminate based on infection status.
- Allow individuals to travel across islands over time.

Controllability and Welfare

- Virus exposure *controllable* if possible to discriminate based on health status without loss of output.
- Welfare thms: If virus exposure controllable, FWT and SWT hold.
  - Logic: Externalities local with controllability.
- If virus exposure not controllable, welfare thms typically do not hold.
  - Logic: Externalities global without controllability.
- With global externalities, conventional wisdom may be incorrect.
  - Economic activity may be too low rather than too high.

Literature

- **Epidemiological literature**.
  - Kermack and McKendrick and Walker (1927), Bourouiba et al. (2014), Morawska et al. (2020), Somsen et al. (2020).
- **Literature on local public goods and club goods**.
  - Tiebout (1956), Buchanan (1965), Stiglitz (1982), Cole and Prescott (1997), Ellickson et al. (1999).
- **Search and matching literature**.
  - Moen (1997), Guerrieri et al. (2010), Wright et al. (2021).
- **Econ-epi literature**.
  - Atkeson (2020), Alvarez et al. (2020), Acemoglu et al. (2020), Chari et al. (2020), Glover et al. (2020), Jones et al. (2020).
  - Eichenbaum et al. (2020), Bethune and Korinek (2020), Melosi and Rottner (2020), Toxvaerd and Rowthorn (2020).

Rest of the Talk

1. **With controllability and perfect observability**.
    - Pandemics as local externalities.
2. **Without controllability**.
    - Pandemics as global externalities.
3. **With controllability and imperfect observability**.
    - Pandemics as local externalities.

With Controllability and Perfect Observability

Model

- Discrete-time model, $t = 0, 1, \ldots, T$.
- Continuum of unit mass of workers/agents.
  - Endowed with one unit of time.
  - Can be in one of three health states (types):

$$\eta \in \left\{ \underbrace{S}_{\textit{Susceptible}}, \underbrace{I}_{\textit{Infected}}, \underbrace{R}_{\textit{Recovered}} \right\}$$

  - Masses $\mu_{\eta t}$.
  - Types publicly observable.
- Continuum of *islands*.
  - Island indexed by 0: *home* island in which no production takes place.
  - Other islands: *work* islands in which production takes place.

Islands

Each island is associated with:

- *A production technology*.
  - For home island, no production technology exists.
  - For work islands, one unit of labor generates A units of consumption good if positive measure of workers. (production requires meetings)
- *Islands indexed by wage rates*, $\boldsymbol{w}_t = \{w_{St}, w_{It}, w_{Rt}\}$, with CDF $F(\boldsymbol{w}_t)$.

Firms choose which island to operate in.

- If they operate on $\boldsymbol{w}_t$, they have to pay wage $w_{\eta t}$ to type η.

Agents

- Endowed with one unit of time.
  - $\ell_\eta(\boldsymbol{w}_t)$: labor allocated by η to island $\boldsymbol{w}_t$; $\int \ell_\eta(\boldsymbol{w}_t)\mathrm{d}F(\boldsymbol{w}_t) = 1,\ \forall\eta$.
- Preferences over the final consumption good are given by

$$U(c) = \sum_{t=0}^{T} \beta^t u(c_t).$$

- Infected agents suffer per-period utility cost κ.

Transmission of the Virus

- $S \rightarrow I \rightarrow R$.
- Susceptible agents become infected in the process of production.
  - Production requires meetings between agents.
  - No infections take place on the home island.
- Probability that S agent becomes infected on work island $\boldsymbol{w}_t$:

$$\psi(\lambda_I(\boldsymbol{w}_t)) = \chi\lambda_I(\boldsymbol{w}_t), \qquad \text{where} \quad \lambda_I(\boldsymbol{w}_t) = \frac{\mu_{It}\ell_I(\boldsymbol{w}_t)}{L(\boldsymbol{w}_t)}$$

  and $L(\boldsymbol{w}_t) \equiv \sum_\eta \mu_{\eta t}\ell_\eta(\boldsymbol{w}_t)$ is total labor supply on island $\boldsymbol{w}_t$.
- Important issue concerns beliefs of infection prob. when $L(\boldsymbol{w}_t) = 0$.
- Infected agents recover with probability α.

Transmission of the Virus

- Aggregate masses of agents evolve according to:

$$\underbrace{\mu_{St+1}}_{\substack{\text{Susceptible}\\\text{tomorrow}}} = \underbrace{\mu_{St}}_{\substack{\text{Susceptible}\\\text{today}}} \underbrace{-\mu_{St}\int_{\boldsymbol{w}_t\neq\boldsymbol{w}_0}\ell_S(\boldsymbol{w}_t)\chi\frac{\mu_{It}\ell_I(\boldsymbol{w}_t)}{L(\boldsymbol{w}_t)}\,\mathrm{d}F(\boldsymbol{w}_t)}_{\text{New infections}},$$

$$\underbrace{\mu_{It+1}}_{\substack{\text{Infected}\\\text{tomorrow}}} = \underbrace{(1-\alpha)\mu_{It}}_{\text{Not recovered}} + \underbrace{\mu_{St}\int_{\boldsymbol{w}_t\neq\boldsymbol{w}_0}\ell_S(\boldsymbol{w}_t)\chi\frac{\mu_{It}\ell_I(\boldsymbol{w}_t)}{L(\boldsymbol{w}_t)}\,\mathrm{d}F(\boldsymbol{w}_t)}_{\text{New infections}},$$

$$\underbrace{\mu_{Rt+1}}_{\substack{\text{Recovered}\\\text{tomorrow}}} = \underbrace{\mu_{Rt}}_{\substack{\text{Recovered}\\\text{today}}} + \underbrace{\alpha\mu_{It}.}_{\text{New recoveries}}$$

Matching Technology

- Competitive production firms choose which island to locate in.
  - Let $\gamma(\boldsymbol{w}_t)$ be the mass of firms on island $\boldsymbol{w}_t$.
  - Each firm pays κ_v to enter ($= 0$ for presentation only.)
- Workers and firms on island $\boldsymbol{w}_t$ matched via $M(L(\boldsymbol{w}_t), \gamma(\boldsymbol{w}_t))$.
  - Market tightness $\theta(\boldsymbol{w}_t) \equiv \gamma(\boldsymbol{w}_t)/L(\boldsymbol{w}_t)$.
  - $m_w(\theta(\boldsymbol{w}_t))$: probability that a worker is matched with a firm.
  - $m_f(\theta(\boldsymbol{w}_t))$: probability that a firm is matched with a worker.
- Matched firm/worker produce A units of goods per unit of time.
- Unmatched workers do not produce, but can get infected.

Allocation

$$Z = \left(\underbrace{\boldsymbol{\mu}}_{\text{masses of types}}, \underbrace{\boldsymbol{\ell}}_{\text{labor supply}}, \underbrace{\boldsymbol{c}}_{\text{consumption}}, \overbrace{\underbrace{\boldsymbol{\Theta}}_{\text{market tightness}}, \underbrace{\boldsymbol{\lambda}}_{\text{relative labor supply}}}^{\text{beliefs}} \right)$$

Need θ, λ to deal with off-equilibrium-path problems (0/0).

A feasible allocation satisfies:

$$\sum_{\eta} \mu_{\eta t} c_{\eta t} \leqslant \int_{\boldsymbol{w}_t \neq \boldsymbol{w}_0} \left(\sum_{\eta} \mu_{\eta t} m_w(\theta(\boldsymbol{w}_t)) A \ell_\eta(\boldsymbol{w}_t) - \gamma(\boldsymbol{w}_t) \kappa_v \right) \, dF(\boldsymbol{w}_t),$$

$$\int \ell_\eta(\boldsymbol{w}_t) \, dF(\boldsymbol{w}_t) = 1,$$

$$\lambda_\eta(\boldsymbol{w}_t) = \frac{\mu_{\eta t} \ell_\eta(\boldsymbol{w}_t)}{L(\boldsymbol{w}_t)}, \qquad \forall L(\boldsymbol{w}_t) > 0, \text{ arbitrary otherwise},$$

$$\boldsymbol{\mu}_{t+1} = G(\boldsymbol{\mu}_t).$$

Controllability in the Model

- Extent of virus exposure depends on mix of susceptible and infected agents in an island.
- Productivity same in all islands independent of infection status.
- Virus exposure controllable because any mix of susceptible and infected agents is feasible without loss of output.
  - Example: Feasible to allocate susceptible agents to a separate island, all producing A. Susceptible agents not exposed to virus.

Susceptible Agent's Decision Problem

$$
\begin{aligned}
V_t(S, \mu_t) = \max_{c_{St},\ \ell_S(\boldsymbol{w}_t)} \quad & u(c_{St}) + \beta \int_{\boldsymbol{w}_t \neq \boldsymbol{w}_0} \ell_S(\boldsymbol{w}_t)\left(1 - \psi\left(\lambda_I(\boldsymbol{w}_t)\right)\right) V_{t+1}(S, \mu_{t+1})\, \mathrm{d}F(\boldsymbol{w}_t) \\
& + \int_{\boldsymbol{w}_t \neq \boldsymbol{w}_0} \ell_S(\boldsymbol{w}_t) \psi\left(\lambda_I(\boldsymbol{w}_t)\right) \left[-\kappa + \beta V_{t+1}(I, \mu_{t+1}) \right] \mathrm{d}F(\boldsymbol{w}_t)
\end{aligned}
$$

subject to

$$
c_{St} \leqslant \int_{\boldsymbol{w}_t \neq \boldsymbol{w}_0} \ell_S(\boldsymbol{w}_t) m_w\left(\theta(\boldsymbol{w}_t)\right) w_{St}\, \mathrm{d}F(\boldsymbol{w}_t),
$$

$$
\int \ell_S(\boldsymbol{w}_t)\, \mathrm{d}F(\boldsymbol{w}_t) = 1.
$$

Infected Agent's Decision Problem

$$V_t\left(I, \mu_t\right) = \max_{c_{It}, \ell_I(\boldsymbol{w}_t)} \quad u\left(c_{It}\right) - \kappa + \alpha\beta V_{t+1}\left(R, \mu_{t+1}\right) + (1-\alpha)\,\beta V_{t+1}\left(I, \mu_{t+1}\right)$$

subject to

$$c_{It} \leqslant \int_{\boldsymbol{w}_t \neq \boldsymbol{w}_0} \ell_I(\boldsymbol{w}_t)\, m_w\left(\theta(\boldsymbol{w}_t)\right) w_{It}\, \mathrm{d}F(\boldsymbol{w}_t),$$

$$\int \ell_I(\boldsymbol{w}_t)\, \mathrm{d}F(\boldsymbol{w}_t) = 1.$$

Recovered Agent's Decision Problem

$$V_t(R, \mu_t) = \max_{c_{Rt}, \ell_R(\boldsymbol{w}_t)} \quad u(c_{Rt}) + \beta V_{t+1}(R, \mu_{t+1})$$

subject to

$$c_{Rt} \leqslant \int_{\boldsymbol{w}_t \neq \boldsymbol{w}_0} \ell_R(\boldsymbol{w}_t) m_w(\theta(\boldsymbol{w}_t)) w_{Rt} \, \mathrm{d}F(\boldsymbol{w}_t),$$

$$\int \ell_R(\boldsymbol{w}_t) \, \mathrm{d}F(\boldsymbol{w}_t) = 1.$$

Competitive Equilibrium

Define the set of active islands by

$$\Gamma_t = \left\{ \boldsymbol{w}_t \ : \ \ell_\eta(\boldsymbol{w}_t) > 0 \ \text{ for some } \eta \in \{S, I, R\} \right\}.$$

A CE is an allocation Z, values, and a set of active islands such that:

1. Agents optimize.
2. $m_f(\theta(\boldsymbol{w}_t)) \sum_\eta \lambda_\eta(\boldsymbol{w}_t)\left(A - w_{\eta t}\right) \leqslant 0$ for all $\boldsymbol{w}_t$ ($=$ if $\boldsymbol{w}_t \in \Gamma_t$).
3. For any $\boldsymbol{w}_t \in \Gamma_t, \lambda_\eta(\boldsymbol{w}_t)$ defined as before.
4. Laws of motion for state $\boldsymbol{\mu}_t$.
5. $\lim_{t\to\infty} \ \beta^t V_t(\eta, \boldsymbol{\mu}_t) \to 0$ for all η.
6. Two refinements.

Refinements To Discipline Off-Equilibrium-Path Beliefs

For any $\boldsymbol{w}_t \in \Gamma_t^c$,

1. If $A - w_{\eta t} > 0$ for all η, then $m_f\left(\theta(\boldsymbol{w}_t)\right) = 0$ and $m_w\left(\theta(\boldsymbol{w}_t)\right) = 1$.

2. If $\hat{V}_t\left(\boldsymbol{w}_t, \eta, \boldsymbol{\mu}_t; \hat{\boldsymbol{\lambda}}_t\right) < V_t\left(\eta, \boldsymbol{\mu}_t\right)$ for all $\hat{\boldsymbol{\lambda}}_t$, then $\lambda_\eta(\boldsymbol{w}_t) = 0$, where

$$\hat{V}_t\left(\boldsymbol{w}_t, S, \boldsymbol{\mu}_t; \hat{\boldsymbol{\lambda}}_t\right) = u\left(c_{St}\right) + \ell_S(\boldsymbol{w}_t)\psi\left(\hat{\lambda}_{It}\right)\left[-\kappa + \beta V_{t+1}\left(I, \boldsymbol{\mu}_{t+1}\right)\right] + \left(1 - \ell_S(\boldsymbol{w}_t)\psi\left(\hat{\lambda}_{It}\right)\right)\beta V_{t+1}\left(S, \boldsymbol{\mu}_{t+1}\right)$$

is value for S of choosing island $\boldsymbol{w}_t$ given beliefs $\hat{\boldsymbol{\lambda}}_t$.

Similarly for other types: $\hat{V}_t\left(\boldsymbol{w}_t, \eta, \boldsymbol{\mu}_t; \hat{\boldsymbol{\lambda}}_t\right)$.

Equilibrium Characterization

An equilibrium has:

- **Mixing** if there exists $\boldsymbol{w}_t$ with $\ell_S(\boldsymbol{w}_t) > 0$ and $\ell_I(\boldsymbol{w}_t) > 0$.
- **Sorting** if there is no mixing.
- **Cross-subsidization** if there exists some $\boldsymbol{w}_t$ and some η, η' with
  - $\ell_\eta(\boldsymbol{w}_t), \ell_{\eta'}(\boldsymbol{w}_t) > 0$ and
  - $w_{\eta t} < A$ and $w_{\eta' t} > A$.

Proposition

Any CE has sorting, no cross-subsidization, and no unemployment.

Informal argument Proof

Welfare Theorems

In any competitive equilibrium:

- All agents consume A.
- Susceptible agents never get infected.
- Recovered agents can be assigned to any island.

Theorem (FWT)

The competitive equilibrium is Pareto optimal.

Theorem (SWT)

Any PO allocation can be decentralized as a CE with LS taxes/transfers.

Proof

Multiple Occupations and/or Multiple Commodities

- Suppose technology with M different types of labor:

$$Y = Af(L_1, \ldots, L_M).$$

- If probability of infection independent of composition of labor types:
  - Welfare Theorems continue to hold.

- Similar results with multiple commodities.

Efficiency of CE

Two key assumptions drive the efficiency results:

1. Virus exposure controllable.
2. Contracts can be a function of publicly-observed health status.

We now relax these assumptions.

Without Controllability

Controllability and Discrimination

- Suppose there is only one work island (denoted by 1) and a home island (denoted by 0).
  - In the work island, $w_{1\eta t} = A$ for all (η, t).
  - No discrimination restriction.
- Allocation z defined as before, with no discrimination restriction.
- Same definition of CE, with obvious modifications.

Efficiency of Equilibrium

Proposition

In the one work-island model, the CE is inefficient.

▸ Proof

Why?

- *Positive* congestion externalities.
- Positive congestion externalities are relevant for a wide class of infection technologies (also with asymptomatic agents).

▸ Robustness: Infection Technology

Source of Inefficiencies in Static Model

- In competitive equilibrium, susceptible agent's labor supply solves:

$$\max_{\ell_S \in [0,1]} \quad u\left(\ell_S A\right) - \ell_S \chi \lambda_I^*(\ell_S^*)\kappa$$

$$\text{where} \quad \lambda_I^*(\ell_S^*) = \frac{\mu_I}{\mu_S \ell_S^* + \mu_I + \mu_R}.$$

- Social planner solves:

$$max_{\ell_S^* \in [0,1]} \quad u\left(\ell_S^* A\right) - \ell_S^* \chi \lambda_I^*(\ell_S^*)\kappa$$

- **Positive congestion externality**. If all susceptible agents increase labor supply a little bit, reduces infection probability for everyone.

Statics vs. Dynamics

Recap

- Untargeted lockdowns not optimal.
- Economic activity can be too low, not too high.
- Subsidies for working may increase welfare.

Without Perfect Observability

Imperfectly-observable Types

- So far assumed all infected are "symptomatic".
- Extend model to allow for "asymptomatic" agents.
  - Infected agents become symptomatic with probability ϕ.
- Types: $\eta \in \{U_S, U_I, I, R\}$.
  - U_S: unknown susceptible.
  - U_I: unknown infected (asymptomatic).
  - U_S and U_I cannot be distinguished, refer as U type.

    $\Longrightarrow$ Must receive the same allocation.
  - R types can be identified even if previously asymptomatic.
- Evolution of types ▸ Equations.

Model with Imperfect Observability

- Static model with risk-neutral agents for presentation.
- Equilibrium definition similar to perfect-observability model.

Equilibrium Characterization with Imperfect Observability

In any competitive equilbrium:

- U and R mix.
- I agents on their own.

Characetization Details: A Pareto Problem

- In any Pareto problem, I separated from U.
- Consider the following Pareto problem:
  - All known infected assigned to island 1, consume A.
  - U types get utility V_U.
  - Trace out the frontier by maximizing welfare of recovered.

Pareto Problem

$$V_R(V_U) = \max_{\{c_\eta, \ell_\eta, \tilde{\pi}_\eta\}} c_R$$

subject to

$$\sum_{\eta \in \{U,R\}} \tilde{\pi}_\eta \left[A \int_{\boldsymbol{w} \neq \boldsymbol{w}_0} \ell_\eta(\boldsymbol{w}) \mathrm{d}F(\boldsymbol{w}) - c_\eta \right] \geqslant 0$$

,

$$c_U - \int_{\boldsymbol{w} \neq \boldsymbol{w}_0} \left[\ell_U(\boldsymbol{w}) 1_{\{\eta = U_S\}} \psi(\lambda_I(\boldsymbol{w})) \kappa - 1_{\{\eta = U_{I,I}\}} \kappa \right] \mathrm{d}F(\boldsymbol{w}) \geqslant V_U.$$

- Market clearing, $\tilde{\pi}_\eta = \mu_\eta$, determines V_U.

Mixing and Efficiency of Equilibrium

Proposition

Any Pareto optimal allocation has mixing of U and R types. ‣ *Proof*

Efficiency of equilibrium.

- $\underline{V}(\eta)$: max value that type η receives on its own ‣ Equations

Proposition

There exists a CE that is efficient and solves the Pareto problem. This CE has cross-subsidization from U to R agents.

$$V_U^* > \underline{V}(U), \quad V_R(V_U^*) > \underline{V}(R).$$

‣ Informal argument ‣ Proof sketch

Infection Probability and Mass of R Agents

- Infection probability decreasing in mass of R agents:

$$\psi(\lambda_I^*(\boldsymbol{w}^*)) = \chi \frac{\mu_{U_I} \ell_U(\boldsymbol{w}^*)}{\mu_U \ell_U(\boldsymbol{w}^*) + \mu_R}.$$

- Result implies social value of vaccines greater than private value.

- Results robust to private information.

Details

Conclusion

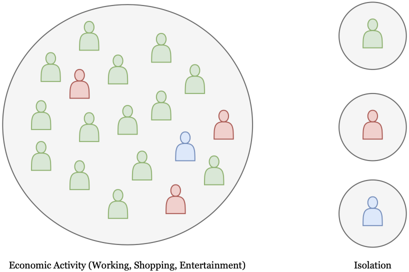
Conclusion

1. With controllability, welfare theorems hold.
    - Lockdowns not needed.
2. Without controllability, CE not efficient.
    - Inability to discriminate key for inefficiency.
    - Conventional wisdom wrong: economic activity in CE too low.
3. With imperfect observability, welfare theorems hold.
    - CE features cross-subsidization.
    - Robust to private information.

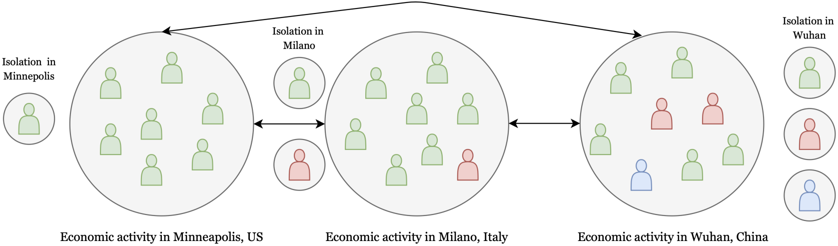
Thank You!

Extra Slides

Global vs. Local Externality View of Pandemics



Typical Economic Approach



Our Approach

Example of Typical Economic Approach

- Eichenbaum, Rebelo and Trabandt (2020):

$$\text{New infections } = \underbrace{\pi_C (S_t C_t^S)(I_t C_t^I)}_{\substack{\text{Infections from} \\ \text{consumption}}} + \underbrace{\pi_N (S_t N_t^S)(I_t N_t^I)}_{\substack{\text{Infections from} \\ \text{work}}} + \underbrace{\pi_O S_t I_t}_{\substack{\text{Random} \\ \text{infections}}}.$$

Their notation:

  - π_i: Infectivity rate in activity $i \in \{C, N, O\}$.
  - S_t, I_t: Masses of susceptible and infected workers.
  - C_t^i: Consumption expenditures by worker of type $i \in \{S, I\}$.
  - N_t^i: Hours worked by worker of type i.

With $\pi_C = \pi_N = 0$ and $\pi_O = \beta$, this model nests standard SIR.

- Similar approaches used elsewhere in the econ-epi literature.

Our Approach

- Anticipating elements of our environment:

$$\text{New infections}_j = \underbrace{\ell_{jS}}_{\substack{\text{labor supply} \\ \text{of type } S}} \times \underbrace{\psi(\lambda_{jI})}_{\substack{\text{infection prob.} \\ \text{in location } j}}$$

$$\psi(\lambda_{jI}) = \underbrace{\chi}_{\substack{\text{infectivity} \\ \text{rate}}} \times \underbrace{\frac{\ell^*_{jI} I}{\ell^*_{jS} S + \ell^*_{jI} I + \ell^*_{jR} R}}_{\text{relative mass of } I \text{ workers}}$$

where $\ell^*_{j\iota}$ denotes equilibrium labor supply of type $\iota \in \{S, I, R\}$ at j.

- Notice the difference between the two approaches:
  - We embrace the local-externality view of pandemics.
  - We model more carefully the infection process.

Back

Proposition 1: Informal Argument

Proposition

Any CE is separating, has no cross-subsidization, and no unemployment.

Informal argument.

- Competition and worker mobility imply that $w_{j\eta t} = A$.
- If there is mixing
  - S agents will strictly prefer island with slightly lower wage.
  - Refinement 1: agents match with probability one.
  - Refinement 2: infected agents will never show up in such islands.
- Competition and free entry ensures no unemployment.

Back

Proof of Proposition 1 (Contradiction + Backward Induction)

Consider the final period T.

- **No cross-subsidization**.
  1. Show that $w_{jIT} \geqslant A$, $\forall j \in \Gamma_T : \ell_{jIT} > 0$. Suppose not. Then, $\exists j \in \Gamma_t : w_{jIt} < A$. Now consider $j' \in \Gamma_t^c : w_{j'IT} > w_{jIT}$ and $w_{j'\eta T} < A$, $\forall \eta$. From eq. condition 6), $m_w(\theta_{j'T}) = 1$ so $\eta = I$ str. better off at j' than at j, a contradiction.
  2. Similar argument establishes $w_{jRT} \geqslant A$ for all $j \in \Gamma_T$.
  3. Use $(1) + (2)$ to show $w_{jST} \geqslant A$, $\forall j \in \Gamma_T : \ell_{jST} > 0$. Suppose not. Consider $j' \in \Gamma_t^c : w_{j'ST} > w_{jST}$ and $w_{j'\eta T} < A$, $\forall \eta$. From eq. condition 6), $m_w(\theta_{j'T}) = 1$. From eq. condition 7), $\ell_{jIT} = 0$. Thus, $\psi(\lambda_{j'IT}) = 0$. $\Longrightarrow S$ str. better off at j', a contradiction.

- **No unemployment**.
  1. Suppose $\exists j \in \Gamma_T : m_w(\theta_{jT}) < 1$ and $\ell_{jST} > 0$. Consider $j' \in \Gamma_t^c : m_w(\theta_{jT}) w_{jST} < w_{j'ST} < A$ and $w_{j'\eta' T} < m_w(\theta_{jT}) w_{j\eta' T}$, $\forall \eta'$. By eq. condition 7), $\psi(\lambda_{j'IT}) = 0$. By eq. condition 6, $m_w(\theta_{j'T}) = 1$, so S better off by switching to j', a contradiction.

Proof of Proposition 1 (Cont.)

- **No mixing**.
  1. Suppose $\exists j \in \Gamma_T : \ell_{jIT}, \ell_{jST} > 0$. Consider $j' \in \Gamma_t^c : w_{j'\eta T} < w_{j\eta T}$ for all η and that $w_{jST} - \psi(\lambda_{jIT})\kappa < w_{j'ST}$. By eq. condition 6), $m_w(\theta_{j'T}) = 1$. By eq. condition 7), $\lambda_{j'IT} = 0$. Hence, S strictly better off by switching to j', a contradiction.

- No cross-subsidization, no unemployment and no mixing imply that $V_T(S, \mu_T) \geqslant V_T(I, \mu_T)$ for all μ_T.

- Next, consider $T-1$. Use the monotonicity result for V and repeat all arguments above to show the same is true.

- Use backward induction to show that this is true for $T-2, \ldots, 0$.

■

Back

Proof of SWT

Some notation:

- $h_t = (\eta_0, \ldots, \eta_t)$: individual agent's t-history.
- $H_t = (\boldsymbol{\mu}_t, \boldsymbol{\gamma}_{t-1}, H_{t-1})$: aggregate t-history.
- Individual allocation rule: $z_t(h_t) = \big(c_t(h_t), \boldsymbol{\ell}_t(h_t)\big)$.
- Firm allocation rule: $\boldsymbol{\gamma}_t(H_t)$.
- Probability distributions over histories:

$$\pi_{t+1}(h_t, S) = \pi_t(h_{t-1}, S)\left(1 - \int_{j\neq 0} \ell_{jt}(h_{t-1}, S)\chi\lambda_{jIt}\right)\,\mathrm{d}j,$$

$$\pi_{t+1}(h_{t-1}, S, I) = \pi_t(h_{t-1}, S)\int_{j\neq 0} \ell_{jt}(h_{t-1}, S)\chi\lambda_{jIt}\,\mathrm{d}j,$$

$$\pi_{t+1}(h_{t-1}, I, I) = (1-\alpha)\pi_t(h_{t-1}, I)$$

$$\pi_{t+1}(h_{t-1}, I, R) = \alpha\pi_t(h_{t-1}, I)$$

$$\pi_{t+1}(h_{t-1}, R, R) = \pi_t(h_{t-1}, R).$$

Proof of SWT (Cont.)

Given some utility levels $\left(\underline{V}(I), \underline{V}(R)\right)$, any PO allocations solves:

$$\max \quad \sum_{t \geqslant 0} \beta^t \sum_{h_t} \pi_t\left(h_t \mid S\right)\left[u\left(c_t\left(h_t \mid S\right)\right) - \mathbf{1}_{\{\eta_t = S\}}\left(\int_{j \neq 0} \ell_{jt}\left(h_t \mid S\right) \psi\left(\lambda_{jIt}\right) \mathrm{d}j\right) \kappa - \mathbf{1}_{\{\eta_t = I\}} \kappa\right]$$

subject to

$$\sum_{t \geqslant 0} \beta^t \sum_{h_t} \pi_t\left(h_t \mid \eta_0\right)\left[\int_{j \neq 0} \ell_{jt}\left(h_t \mid \eta_0\right)\left[u\left(c_t\left(h_t \mid \eta_0\right)\right) - \mathbf{1}_{\{\eta_t = I\}} \kappa\right]\right] \geqslant \underline{V}\left(\eta_0\right), \quad \eta_0 \in \{I, R\}$$

$$\sum_{h_t} \pi_t\left(h_t \mid h_0\right) c_t\left(h_t\right) \leqslant \sum_{h_t} \pi_t\left(h_t \mid h_0\right)\left[\int_{j \neq 0} m_w\left(\theta_{jt}\right) A \ell_{jt}\left(h_t\right) \mathrm{d}j\right],$$

$$\int \ell_{jt}\left(h_t\right) \mathrm{d}j = 1,$$

Probability distributions over histories,

where:

$$\lambda_{jIt} = \frac{\sum_{h_t} \pi_t\left(I, z_{t-1}, h_{t-1} \mid h_0\right) \mu_{It} \ell_{jt}\left(I, z_{t-1}, h_{t-1} \mid h_0\right)}{\sum_{\eta} \sum_{h_t} \pi_t\left(\eta, z_{t-1}, h_{t-1} \mid h_0\right) \mu_{\eta t} \ell_{jt}\left(\eta, z_{t-1}, h_{t-1} \mid h_0\right)}.$$

Proof of SWT (Cont.)

1. Using similar arguments to Prop. 1, establish that allocations where any S gets infected are dominated by allocations where they don't. (Assign S agents to an otherwise identical island with no I types).

   It follows that no S gets infected in a PO allocation (same as in CE).

2. Since productivity is greater in islands $j > 0$, no individual placed on island $j = 0$. Since $\kappa_v = 0$, the planner can always assign enough firms to any island so that $m_w(\theta_{jt}) = 1$ for all $j \in \Gamma_t$.

   Hence, no unemployment in a PO allocation (same as in CE).

3. Now, pick any feasible levels of consumption $\{c_t(h_t)\}$.

4. By appropriately choosing LS tax/transfers, the result follows.

∎

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Evolution of Histories

$$\pi_{t+1}(h_t, S) = \pi_t(h_{t-1}, S)\left(1 - \ell_t(h_{t-1}, S)\chi\lambda_{It}\right)$$

$$\pi_{t+1}(h_{t-1}, S, I) = \pi_t(h_{t-1}, S)\,\ell_t(h_{t-1}, S)\chi\lambda_{It}$$

$$\pi_{t+1}(h_{t-1}, I, I) = (1-\alpha)\,\pi_t(h_{t-1}, I)$$

$$\pi_{t+1}(h_{t-1}, I, R) = \alpha\pi_t(h_{t-1}, I)$$

$$\pi_{t+1}(h_{t-1}, R, R) = \pi_t(h_{t-1}, R).$$

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Proof of Proposition 4

No cross-subsidization.

1. Define firm profits associated with each type η as:

$$\Pi_t(\eta) \equiv \mu_{\eta t} \times \left[\ell_{\eta t} A - c_{\eta t}\right].$$

2. Since there is perfect competition, we have $\sum_{\eta} \mu_{\eta t} \Pi_t(\eta) = 0$.

3. Next, we show that $\Pi_t(\eta) = 0$ for each η. Suppose not. Then $\exists \eta : \Pi_t(\eta) > 0$. This implies $\exists \hat{\eta}$ s.t. $\ell_{\hat{\eta} t} A - c_{\hat{\eta} t} > 0$. Consider a deviating firm offering:

$$\begin{aligned} \tilde{c}_{\eta t} &= c_{\eta t}, \qquad \forall \eta \neq \hat{\eta}, \\ \tilde{c}_{\hat{\eta} t} &= c_{\hat{\eta} t} + \varepsilon, \end{aligned}$$

where $0 < \varepsilon < \ell_{\hat{\eta} t} A - c_{\hat{\eta} t}$ and $\tilde{c}_{\eta t} = 0$ for all η. Therefore, the deviating firm makes strictly positive profits, a contradiction.

Proof of Proposition 4 (Cont.)

I **and** R **supply 1 unit of labor in the work island in all periods**.

1. Suppose $\ell_{It} < 1$ for some t. By increasing ℓ_{It}, the I type can increase its utility while leaving the infection cost unchanged. Hence, $\ell_{It} < 1$ contradicts optimality.

   This result + no cross-subsidization imply $c_{It} = A$ for all t.

2. Identical argument for $\eta = R$.

Proof of Proposition 4 (Cont.)

Mixing.

1. Suppose $\ell_{St} = 0$ for all t. By no cross-subsidization, $c_{St} = 0$ for all t and firm makes zero profits.

2. Consider:

$$\tilde{\ell}_{S0} = \varepsilon > 0 \qquad \text{and} \qquad \tilde{c}_{S0} = \varepsilon A.$$

Clearly, firm continues to make 0 profits. Change in welfare for S:

$$\Delta\mathcal{W}(S) = \underbrace{u(\varepsilon A) - \varepsilon\psi(\lambda^*_{It})\kappa + \beta\left[1 - \varepsilon\psi(\lambda^*_{It})\right]V_1(S,S) + \beta\varepsilon\psi(\lambda^*_{It})V_1(S,I)}_{\text{utility with some mixing}}$$

$$- \underbrace{\left[u(0) - \beta V_1(S,S)\right]}_{\text{utility with no mixing}}.$$

Proof of Proposition 4 (Cont.)

3. Differentiating above expression wrt ε and evaluating at $\varepsilon = 0$:

$$u'(0) - \psi(\lambda_{It}^*)\kappa + \psi(\lambda_{It}^*)\beta\left[V_1(S,I) - V_1(S,S)\right].$$

4. Note that under the original allocation:

$$V_1(S,S) = \frac{1-\beta^T}{1-\beta}u(0), \quad \text{and} \quad V_1(S,I) \geqslant \frac{1-\beta^T}{1-\beta}\left[u(A) - \kappa\right].$$

   Therefore, the above derivative is bounded from below by:

$$u'(0) - \kappa + \psi(\lambda_{It}^*)\beta\left[\frac{1-\beta^T}{1-\beta}\left(u(A) - \kappa - u(0)\right)\right].$$

   Since $u(A) - \kappa > u(0)$, if $u'(0) - \kappa > 0$, this alternative allocation makes S str. better off.

Proof of Proposition 4 (Cont.)

5. If $u(A) - \kappa < u(0)$, then if
$$u'(0) - \kappa + \beta \left[\frac{1-\beta^T}{1-\beta} \big(u(A) - \kappa - u(0) \big) \right] > 0,$$
S agents str. better off.

6. Thus, a sufficient condition for S to be str. better off under such an alternative allocation is:
$$u'(0) - \kappa > \Omega \equiv \max \left\{ \beta \left[\frac{1-\beta^T}{1-\beta} \big(u(A) - \kappa - u(0) \big) \right], 0 \right\}.$$

7. Under this assumption, S-type agents are strictly better of by mixing, which is a contradiction.

   It then follows that there is mixing in at least one period.

■

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One work-island model: Efficiency of Equilibrium

Proposition

In the one work-island model, the CE is inefficient.

Proof.

1. Compare (susceptible and infected) agents' problems with the Pareto problem, restricted to the one work-island case.
2. Note that planner internalizes the effects of the labor allocation on the infection probability $\psi(\lambda_{It})$, but individuals do not.

■

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Generalized Infection Technology

- Consider the following generalization of our infection technology:

$$\chi \frac{\mu_{It}\ell_{It}}{\left(\mu_{St}\ell_{St} + \mu_{It}\ell_{It} + \mu_{Rt}\ell_{Rt}\right)^{2-\vartheta}}.$$

- This technology is similar to that of Acemoglu et al. (2020), and it nests several special cases:
  - $\vartheta = 1$: our baseline.
  - $\vartheta = 2$: standard economic-SIR.
- Here, $\vartheta \in [1,2]$ governs the returns to scale in meetings and can play a key role in the study of externalities:
  - $\vartheta = 1 \Longrightarrow$ CRS.
  - $\vartheta \in (1,2] \Longrightarrow$ IRS.

Generalized Infection Technology

$$\chi \frac{\mu_{It}\ell_{It}}{\left(\mu_{St}\ell_{St} + \mu_{It}\ell_{It} + \mu_{Rt}\ell_{Rt}\right)^{2-\vartheta}}.$$

- With $\vartheta = 1$, the probability that a particular susceptible agent gets infected is mediated by the presence of R and S agents.
- With $\vartheta = 2$, there is no notion of herd immunity or positive congestion effects.
- A desirable feature of $\vartheta \in (1, 2]$ is that it captures the idea that more meetings take place in more densely-occupied areas.
- No consensus on which technology is more appropriate.
  - Empirically-relevant estimates of ϑ likely in between 1 and 2.

We now study how our results depend on ϑ.

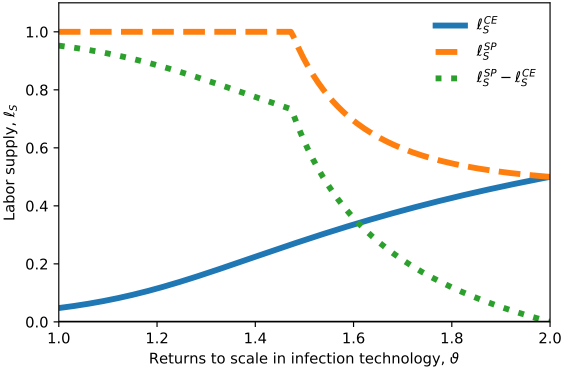
Robustness: Local-Externality View of Pandemics

- We now make explicit our (previous) assumption that production requires a positive mass of agents $\underline{L}$.
  - Otherwise, with IRS infection technologies in a multi-island setup, equilibrium may fail to exist.

- **Proposition**. Any CE is efficient.
  - Representative firm allocates workers to $J+1$ work islands.
  - J pinned down by $\underline{L}$.
  - Separation of I types from the rest.
  - $\pi(U)\ell_U/J$ and $\pi(R)/J$ of U and R workers allocated to each island.
  - Firm problem identical to Pareto problem.

Robustness: Global-Externality View of Pandemics (SIR)

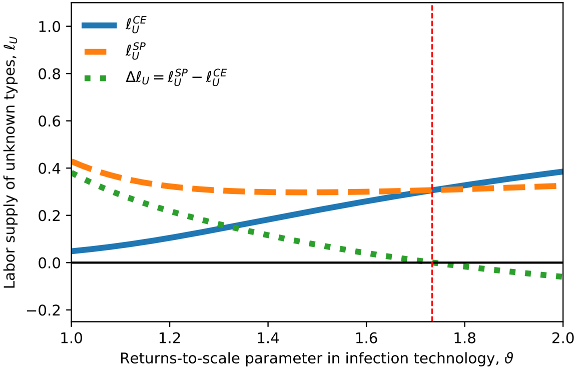
- In SIR model, with log preferences, CE is inefficient and the result that aggregate economic activity is too low is robust to $\vartheta \in [1,2)$.



- $\vartheta = 2$ misses positive congestion externalities (existing literature).

Robustness: Global-Externality View of Pandemics (UIR)

- In UIR model, CE is inefficient, but whether aggregate economic activity is too high or too low can depend on $\vartheta \in [1,2]$.



- With $\vartheta = 2$, typically "too much" economic activity (consistent with existing literature); for most ϑ values, opposite is true.

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Statics vs. Dynamics

- In static model, susceptible agents always work too little.
  - By working more, S reduce infection probability for other S agents.
- In dynamic model, additional externality.
  - By increasing its labor supply, S increase flow of newly infected.
  - Increases probability of future infection.
  - Race between the static and dynamic externalities.

Dynamic Model

- Only interesting problem is that of susceptible agents, which is:

$$V_t\left(S, \mu_t, \lambda_{It}^*\right) = \max_{c_{St},\ \ell_{St}} \quad u\left(c_{St}\right) + \beta \ell_{St}\left[1 - \psi\left(\lambda_{It}^*\right)\right] V_{t+1}\left(S, \mu_{t+1}, \lambda_{It+1}^*\right)$$
$$+ \ell_{St}\psi\left(\lambda_{It}^*\right)\left[-\kappa + \beta V_{t+1}\left(I, \mu_{t+1}, \lambda_{It+1}^*\right)\right]$$
$$\text{s.t.} \quad c_{St} \leqslant \ell_{St} A,$$

Don't internalize effect of ℓ_S on current *and* future infection prob.

- FOC:

$$u'(c_{St})A - \psi(\lambda_{It}^*)\kappa - \beta\psi(\lambda_{It}^*)\left\{\frac{\partial V_{t+1}(S)}{\partial \mu_{St+1}} + \frac{\partial V_{t+1}(S)}{\partial \mu_{It+1}}\right\} = 0.$$

Effect of Small Increase in Labor Supply of S

Total derivative wrt ℓ_S evaluated at equilibrium allocation:

$$\underbrace{\underbrace{-\ell^*_{St}\psi'(\lambda^*_{It})\frac{\partial\lambda^*_{It}}{\partial\ell^*_S}\kappa}_{\text{static component }(+)}+\underbrace{\beta\left[\frac{\partial V_{t+1}(S)}{\partial\mu_{It+1}}-\frac{\partial V_{t+1}(S)}{\partial\mu_{St+1}}\right]\ell^*_{St}\chi\frac{\partial\lambda^*_{It}}{\partial\ell^*_S}}_{\text{dynamic component }(+/-)}}_{\text{externality from current infection}}$$

$$+\quad\underbrace{\beta\frac{\partial V_{t+1}(S)}{\partial\lambda^*_{It+1}}\frac{\partial\lambda^*_{It+1}}{\partial\ell^*_S}}_{\text{externality from future infection }(-)}$$

Two Externalities in Dynamic Model

Externality from current infection.

- Static component identical to static model.
  - Always positive.
- Dynamic component due to change in future masses of types.
  - Typically positive.

Externality from future infection.

- Increasing ℓ_S increases the *flow* of newly infected agents $\mu_{St}\ell_S\lambda^*_{It}$.
- This increases infection probability in the future.
- Negative externality.

Overall effect on welfare ambiguous.

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Type Transitions with Asymptomatic Agents

$$\pi_{t+1}\left(h_{t-1},U_S,U_S\right)=\pi_t\left(h_{t-1},U_S\right)\left[1-\int_{j\neq 0} l_{jt}\left(h_{t-1},U\right)\chi\lambda_{Ijt}\mathrm{d}j\right],$$

$$\pi_{t+1}\left(h_{t-1},U_S,U_I\right)=\underbrace{\left(1-\phi\right)}_{\text{asymptomatic}}\underbrace{\pi_t\left(h_{t-1},U_S\right)\int_{j\neq 0}\ell_{jt}\left(h_{t-1},U\right)\chi\lambda_{Ijt}\mathrm{d}j}_{\text{new infections}},$$

$$\pi_{t+1}\left(h_{t-1},U_S,I\right)=\phi\pi_t\left(h_{t-1},U_S\right)\int_{j\neq 0}\ell_{jt}\left(h_{t-1},U\right)\chi\lambda_{Ijt}\mathrm{d}j$$

$$\pi_{t+1}\left(h_{t-1},U_I,U_I\right)=\underbrace{\left(1-\phi\right)\left(1-\alpha\right)}_{\substack{\text{Asymptomatic}\\ \text{who not recover}}}\pi_t\left(h_{t-1},U_I\right),$$

$$\pi_{t+1}\left(h_{t-1},U_I,I\right)=\phi\left(1-\alpha\right)\pi_t\left(h_{t-1},U_I\right),$$

$$\pi_{t+1}\left(h_{t-1},U_I,R\right)=\alpha\pi_t\left(h_{t-1},U_I\right),$$

$$\pi_{t+1}\left(h_{t-1},I,R\right)=\alpha\pi_t\left(h_{t-1},I\right),$$

$$\pi_{t+1}\left(h_{t-1},R,R\right)=\pi_t\left(h_{t-1},R\right).$$

Mixing of U and R types

Proposition

Any Pareto optimal allocation has mixing of U and R types.

Proof. Notice that:

$$\underbrace{\chi \frac{\sum_{h_{t-1}} \pi_t\left(h_{t-1}, U_l\right) \ell_{jt}\left(h_{t-1}, U_l\right)}{\sum_{h_{t-1}} \sum_{\eta \neq l,R}\left[\pi_t\left(h_{t-1}, \eta\right) \ell_{jt}\left(h_{t-1}, \eta\right)\right]}}_{\equiv \psi(\lambda_{jlt}; U)} > \underbrace{\chi \frac{\sum_{h_{t-1}} \pi_t\left(h_{t-1}, U_l\right) \ell_{jt}\left(h_{t-1}, U_l\right)}{\sum_{h_{t-1}} \sum_{\eta \neq l}\left[\pi_t\left(h_{t-1}, \eta\right) \ell_{jt}\left(h_{t-1}, \eta\right)\right]}}_{\equiv \psi(\lambda_{jlt}; U, R).}$$

- U willing to give some consumption to pool with R.
- Mix U and R.
- Redistribute from U to R suitably to make both types weakly better off, with strict inequality for at least one of them.

■

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Autarky Values

For U types:

$$\underline{V}(U) = \max \sum_{t,h_t} \beta^t \pi(h_t \mid U) \left[u\big(c_t(h_t)\big) - \ell_t(h_t) \mathbf{1}_{\{\eta_t = U_S\}} \psi(\lambda_{It}) \kappa - \mathbf{1}_{\{\eta_t = U_I\}} \kappa \right]$$

subject to

$$\sum_{h_t} \pi(h_t \mid U) \left[c_t(h_t \mid U) - \ell_t(h_t \mid U) A \right] \leqslant 0, \quad \forall t,$$

$$\lambda_{It} = \frac{\sum_{h_{t-1}} [\pi_t(h_{t-1}, U_I) \ell_t(h_{t-1}, U)]}{\sum_{h_{t-1}} \sum_{\eta = U,R} \left[\pi_t(h_{t-1}, \eta) \ell_t(h_{t-1}, \eta) \right]}.$$

For R types:

$$\underline{V}(R) = \sum_{t=0}^{T} \beta^t u(A).$$

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Efficiency of Equilibrium

Proposition

There exists a CE that is efficient and solves the Pareto problem. This CE has cross-subsidization from initial U to initial R agents.

$$V_U^* > \underline{V}(U), \quad V_R(V_U^*) > \underline{V}(R).$$

- Initial R agents receive consumption $>$ marginal product.
- Initial U agents receive consumption $<$ marginal product.
- R valuable to initial U agents since lower infection prob.
- U agents willing to give up consumption to pool with them.

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Efficiency of CE with Asymptomatic Agents

1. Show: (a) $V_R(V_U)$ is a decreasing function, (b) $V_R(\underline{V}(U)) > \underline{V}(R)$, and (c) $\lim_{V_U \to \infty} = \sum_{t=0}^{T} \beta^t u(0)$.
   (a) Follows from inspection of SPP.
   (b) Suppose $V_U = \underline{V}(U)$. Redistribute from U to R.
   (c) Follows from inspection of SPP.

2. **Existence**.
   - In any CE, the best response in terms of relative proportions of initial U and R agents, $\rho(V_U) = \tilde{\pi}_0(U; V_U)/\tilde{\pi}_0(R; V_U)$ has a fixed point at relative population proportion $\pi_0(U)/\pi_0(R)$.
   - Consider the firm's programming problem with market utilities $\left(V_U, V_R(V_U), \underline{V}_I\right)$, and show that $\tilde{\rho}(V_U)$, the relative proportion that solves this problem, and show that if $V_U < \underline{V}_U$, $\tilde{\rho}(V_U) = \infty$ and $\tilde{\rho}(V_U) = 0$ as $V_U \to \infty$.

Efficiency of CE with Asymptomatic Agents

2. **Existence** (cont.).
   - Since $\tilde{\rho}(V_U)$ is continuous, $\exists V_U^* : \tilde{\rho}(V_U^*) = \pi_0(U)/\pi_0(R)$.
   - At this $\tilde{\rho}(V_U^*)$, the Pareto problem implies firms make zero profits.
   - No individual firm can profitably deviate $\Longrightarrow$ eq. contract.
3. **Efficiency**. Note that CE outcomes solve the Pareto problem.

■

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Robustness to Private Information

- Suppose R types are publicly known, but the other types are private.
- Competitive equilibrium coincides with the earlier one.
  - R types get paid more than their marginal product.
  - U types get paid less than their MP.
  - I types get paid their MP.
  - No type has incentives to mimic any other type.

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