Practice Questions for Midterm Exam 1

ECO 3302 – Intermediate Macroeconomics

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March 5, 2025

1. Consider a country that experienced 2% real output growth and no growth in hours worked in 2024. What is labor-productivity growth for this country? Show your derivations.

2. Suppose nominal GDP increased by 5% in 2024. Assuming that inflation was 3%, what was real GDP growth? Show your derivations.

3. Consider the following production functions:

$$
\begin{aligned}
&Y = AK, \qquad \text{where } A > 0,\\
&Y = AK^{\alpha}L^{1-\alpha}, \qquad \text{where } A > 0,\ \alpha \in (0,1)\\
&Y = AK^{\alpha}L^{\beta}, \qquad \text{where } A > 0,\ \alpha, \beta \in (0,1) \text{ and } \alpha + \beta < 1
\end{aligned}
$$

   (a) Calculate the returns to scale for each function.

   (b) Derive the marginal products of capital and labor for each function.

   (c) Derive the second-order derivatives, direct ones and cross partials, for each function.

   (d) Check the Inada conditions for each function.

   (e) Are K and L in the last two production functions q-complements?

4. Derive the real wage and the real rental rate of capital for a perfectly competitive economy with a representative firm operating production functions:

$$
\begin{aligned}
&Y = AKL, \qquad \text{where } A > 0,\\
&Y = AK^{\alpha}L^{1-\alpha}, \qquad \text{where } A > 0,\ \alpha \in (0,1)\\
&Y = AK^{\alpha}L^{\beta}, \qquad \text{where } A > 0,\ \alpha, \beta \in (0,1) \text{ and } \alpha + \beta < 1
\end{aligned}
$$

(a) What is the effect of a wave of immigration in the real wage? And in the rental price of capital?

(b) What is the effect of a war that results in half the capital stock in the real wage? And in the rental rate of capital?

(c) What is the effect of technological progress in the real wage and in the real rental price of capital?

(d) Is the real wage increasing or decreasing in the capital-labor ratio? and the real rental rate of capital?

5. Derive labor, capital, and profit shares for a perfectly competitive economy for the following two technologies:

$$Y = AK^{\alpha}L^{1-\alpha}, \qquad \text{where } A > 0,\ \alpha \in (0,1)$$
$$Y = AK^{\alpha}L^{\beta}, \qquad \text{where } A > 0,\ \alpha, \beta \in (0,1) \text{ and } \alpha + \beta < 1$$

6. There are three sectors in the economy: agriculture, manufacturing, and services. The value added of these sectors is \$50, \$100, and \$250 millions, respectively. In this economy, consumption is \$240 millions, investment is \$60 millions, and government spending \$120 millions. Total payments to labor are \$300 millions, payments to capital are \$60 millions, and profits amount to \$40 millions. Calculate GDP using both the production and the income method. With the current data, you cannot calculate GDP using the expenditure method, but you could back out the quantity that you need to do so. What is this quantity and what does it account for?

7. Consider an economy that produces and consumes bread and eggs:

Good	**Quantity in 2010**	**Price in 2010**	**Quantity in 2020**	**Price in 2020**
Bread	90	3	100	7
Eggs	150	6	165	12

(a) Calculate the percentage change in the price of each good.

(b) Using 2010 as the base year, calculate for each year: nominal GDP, real GDP.

(c) Calculate the percentage change in the GDP deflator.

(d) Using 2010 as the base year, calculate the percentage change in the CPI.

(e) Is the CPI any different than the GDP deflator? Why?

8. Do exercises in WIO2.1: Work It Out—Ch. 2.

9. Consider an economy with:

$$\begin{aligned} Y &= C + I + G \\ Y &= 1,000 \\ G &= 180 \\ T &= 300 \\ C &= 160 + 0.7(Y - T) \\ I &= 400 - 25r \end{aligned}$$

   (a) Compute national savings, public savings, and private savings.
   (b) Find the equilibrium interest rate.
   (c) Suppose government spending is fixed, but taxes decrease to 200. Compute private, public, and national savings. What is the change in consumption?
   (d) Find the new equilibrium interest rate.
   (e) Plot both the old and the new equilibrium in the same graph.

10. Suppose one country grows at an average annual growth rate of 2% per year and another country grows at 3% per year. How much richer would the fast-growing country be in comparison to the slow-growing country if both of them start at the same initial level of GDP after 10, 20, 50, 100 years?

11. Suppose GDP of country A is $3 billions today, and $2 billions 20 years ago. What is the average annual growth rate of this country?

12. Suppose GDP is $3 billions. How long will it take to quadruple its GDP if GDP grows at an average rate of 5% per year?

13. Suppose GDP is $3 billions. What was GDP fifteen years ago if the growth rate is 2% per year?

14. Suppose the economy produces according to: $Y(t) = A(t)K(t)^{0.3}L(t)^{0.7}$.

   (a) Decompose the growth rate of GDP into the contributions of technology, capital, and labor. Show each step of your derivations.
   (b) Suppose GDP grows at 2% per year. Capital grows at 0.5% per year and labor at 1% per year. What is the contribution of technology to GDP growth? And of capital? And of labor?
   (c) Now suppose output is given by $Y(t) = A(t)K(t)^{\alpha}L(t)^{\beta}$. You still want to calculate the contributions of technology, capital, and labor to GDP growth. The problem is you don't know the values of α and β. How would you compute these two objects using data?

Solow's Growth Model

15. The economy of Highland Park produces output according to $Y_t = AK_t^{\alpha}L^{1-\alpha}$, where A is the level of technology, K is the capital stock, and L denotes labor. The citizens of Highland Park save a constant fraction $0 < s < 1$ of their income Y_t, and invest the rest in accumulating physical capital, which in the aggregate evolves according to $K_{t+1} = (1-\delta)K_t + I_t$, where $0 < \delta < 1$ is the depreciation rate of physical capital, and I_t is investment.

RECOMMENDED NOTATION:

- Use upper-case letters to denote aggregate variables:
  * Y: Aggregate output
  * K: Aggregate capital
  * L: Aggregate labor
- Use lower-case letters with tildes to denote per-capita variables:
  * $\tilde{y} = \frac{Y}{L}$: Output per capita
  * $\tilde{k} = \frac{K}{L}$: Capital-labor ratio
- Use g_x to denote the growth rate of variable x. Eg, $g_x = \frac{\Delta x_{t+1}}{x_t}$

Answer the following questions, *clearly showing your derivations*:

(a) Derive income shares in the Solow's model, where markets are competitive and firms sell at price P_t, pay workers wage W_t, and rent capital at rate R_t.

(b) Decompose the growth rate of aggregate output into the contributions of technology, capital, and labor.

(c) Transform the aggregate production function to per-capita terms, $\tilde{y}_t = f(\tilde{k}_t)$.

(d) Make a graph that contains the *per-capita* production function and the investment function. Display the following objects: output, investment, and consumption per capita. Clearly indicate what each axis is.

(e) Transform the aggregate capital's low of motion to per capita terms.

(f) Find the steady-state levels of capital per capita, output per capita, and consumption per capita. What makes a country richer?

(g) Make a graph that shows how steady-state capital is determined. That is, display the investment function and the depreciated-capital function and indicate what the steady-state levels of capital per worker and investment per worker are. Clearly indicate what each axis is.

(h) Find the golden-rule savings rate, s_{gold}.

16. The state of Texas produces output according to $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where A is the level of technology, K is the capital stock, and L denotes labor. Texas residents save a constant fraction $0 < s < 1$ of their income Y_t, and invest the rest in physical capital, which in the aggregate evolves according to $K_{t+1} = (1-\delta)K_t + I_t$, where $0 < \delta < 1$ is the depreciation rate of physical capital, and I_t is investment. Population in TX is expected to increase at rate $n > 0$ each year.

RECOMMENDED NOTATION:

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  - K: Aggregate capital
  - L: Aggregate labor
- Use lower-case letters with tildes to denote per-capita variables:
  - $\tilde{y} = \frac{Y}{L}$: Output per capita
  - $\tilde{k} = \frac{K}{L}$: Capital-labor ratio
- Use g_x to denote the growth rate of variable x. Eg, $g_x = \frac{\Delta x_{t+1}}{x_t}$

Answer the following questions, *clearly showing your derivations*:

(a) Decompose the growth rate of aggregate output into the contributions of technology, capital, and labor.

(b) Transform the aggregate production to per-capita terms, $\tilde{y}_t = f(\tilde{k}_t)$.

(c) Make a graph that contains the *per-capita* production function and the investment function. Display the following objects: output, investment, and consumption per capita. Clearly indicate what each axis is.

(d) Transform the aggregate capital's low of motion to per capita terms.
(*Hint*: Start from $\tilde{k}_t = K_t/L_t$ and use one math "trick" that we learned in class)

(e) Find the steady-state levels of capital per capita, output per capita, and consumption per capita. What's the value of output per capita if $A = 1, \alpha = 1/2, s = 0.2, \delta = 0.08$ and $n = 0.02$? And the one for consumption per capita?

(f) Calculate the growth rate of capital per worker. Is it increasing or decreasing?

(g) Suppose we compare the state of Texas with New Mexico. The two states have the same values for s, n, δ, A, but New Mexico is initially poorer; that is, $0 < \tilde{k}_0^{NM} < \tilde{k}_0^{TX}$. Do NM and TX reach the same steady state? Which state, if any, grows faster? Is there any growth after TX reaches its steady state?

(h) Now suppose TX saves at higher rate than NM: $s^{TX} > s^{NM} > 0$. Do NM and TX reach the same steady state?

17. The US President is worried about the economy and has asked you to analyze the economy before he/she takes any policy decision. He tells you that output in the US is produced according to $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$, where A is the level of technology, K is the capital stock, and L denotes labor. US residents save a constant fraction $0 < s < 1$ of their income Y_t, and invest the rest in physical capital, which evolves according to $K_{t+1} = (1-\delta)K_t + I_t$, where $0 < \delta < 1$ is the depreciation rate, and I_t is investment. US population and technology are expected to grow at rates n and g_A each year, respectively, starting at $L_0 > 0$ and $A_0 > 0$.

RECOMMENDED NOTATION:

- Use upper-case letters to denote aggregate variables:
  - Y: Aggregate output
  - K: Aggregate capital
  - L: Aggregate labor
- Use lower-case letters with tildes to denote per-capita variables:
  - $\tilde{y} = \frac{Y}{L}$: Output per capita
  - $\tilde{k} = \frac{K}{L}$: Capital-labor ratio
- Use lower-case letters with hats to denote variables in effective units:
  - $\hat{y} = \frac{Y}{AL}$: Output in effective units
  - $\hat{k} = \frac{K}{AL}$: Capital in effective units
- Use g_x to denote the growth rate of variable x. Eg, $g_x = \frac{\Delta x_{t+1}}{x_t}$

Answer the following questions, *clearly showing your derivations*:

(a) Decompose the growth rate of aggregate output into the contributions of technology, capital, and labor.

(b) Transform the aggregate production function to per-effective units, $\hat{y}_t = f(\hat{k}_t)$.

(c) Make a graph that contains the *per-effective-worker*: production function, investment function, and break-even capital line. Display the following objects: output, investment, consumption per effective unit, steady-state level of capital in effective units. Clearly indicate what each axis is.

(d) Transform the aggregate capital's low of motion to effective units. (*Hint*: Start from $\tilde{k}_t = K_t/(A_t L_t)$ and use two math "tricks" learned in class)

(e) Find the steady-state levels in of capital and output in effective units.

(f) Find the output per capita along a balanced growth path (BGP). At what rate does output per capita grow?

Now suppose the President tells you that the savings rate is 20%, the depreciation rate is 10%, population growth is 2%, and the rate of labor-augmenting technological progress is 2%.

(g) What would output per capita be in year 30 years from now if the level of technology is now $A = 1$?

(h) Suppose the president is contemplating the following policies:

- Giving Americans incentives to save so that the savings rate reaches 30%.
- Giving Americans incentives to have kids so that pop. growth reaches 3%.
- Giving American firms incentives to build more durable machines so that depreciation decreases to 5%.
- Giving US scientists more funds so that they develop better technologies and labor-augmenting technological progress reaches 5%.

Are all these policies a good idea? If not, which one is not and why?

(i) Which policies, if any, have long-run growth effects? Explain. Which policies, if any, have short-run growth or level effects?

(j) Calculate the effect that each policy would have on output per-capita 30 years from now. In light of these numbers, which policy would you recommend?