Short Technical Note on Lectures 2 & 3

ECO 3302 – Intermediate Macroeconomics

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1 Nominal vs. Real GDP

In class, we saw that there are several ways to compute Gross Domestic Product (GDP). Historically, the two most popular ones have been:

- **Nominal GDP**, which measures the total value of final goods and services produced in an economy using *current* prices during the period in which the output is produced. Formally, nominal GDP in year t can be written as

$$\text{Nominal GDP}_t = \sum_{i=1}^{N} p_{it} q_{it},$$

where i indexes different goods and services (e.g., bread, yogurt, ...), and t indexes time. The variable p_{it} represents the price of good/service i in year t, and q_{it} the quantity of that good or service produced in that same year.

Example: Suppose there are only two goods in the economy, bread and yogurt. Nominal GDP in year 2020 is then calculated as:

$$\begin{aligned}\text{Nominal GDP}_{2020} &= \sum_{i=1}^{2} p_{i,2020} q_{i,2020} \\ &= p_{\text{bread},2020} \cdot q_{\text{bread},2020} + p_{\text{yogurt},2020} \cdot q_{\text{yogurt},2020}.\end{aligned}$$

By inspecting the equation above, we can see that nominal GDP may change over time because prices change, quantities change, or both. In macroeconomics, we are often interested in *real* economic activity (i.e., the behavior of physical quantities).

For that reason, we also measure GDP in *real* terms—that is, we remove the effect of price changes. Historically, this have been done by computing real GDP using *constant* prices, which are also referred to as *base* prices.

- **Real GDP using base prices** measures the total value of final goods and services produced in an economy in a given year using *constant* prices. The basic idea is to fix prices to those observed in a reference year—the *base* year—and then compute GDP at different points in time using those prices. By doing so, we arrive at a measure of GDP that only considers changes in quantities, giving us an idea of how much aggregate output grows over time. Suppose we set year t as the base year. In that case, we compute real GDP using base-year prices as

$$\text{Real GDP}_t = \sum_{i=1}^{N} p_{it} q_{it},$$
$$\text{Real GDP}_{t+j} = \sum_{i=1}^{N} p_{it} q_{it+j},$$

where $j = 1, 2, \ldots$.

Example: Suppose there are only two goods in the economy, bread and yogurt. Assume we set 2020 as the base year. Then, real GDP at different points in time can be calculated using *constant* prices as:

$$\begin{aligned}
\text{Real GDP}_{2020} &= \sum_{i=1}^{2} p_{i,2020} q_{i,2020} = p_{\text{bread},2020} \cdot q_{\text{bread},2020} + p_{\text{yogurt},2020} \cdot q_{\text{yogurt},2020}, \\
\text{Real GDP}_{2021} &= \sum_{i=1}^{2} p_{i,2020} q_{i,2021} = p_{\text{bread},2020} \cdot q_{\text{bread},2021} + p_{\text{yogurt},2020} \cdot q_{\text{yogurt},2021}, \\
&\vdots \\
\text{Real GDP}_{2024} &= \sum_{i=1}^{2} p_{i,2020} q_{i,2024} = p_{\text{bread},2020} \cdot q_{\text{bread},2024} + p_{\text{yogurt},2020} \cdot q_{\text{yogurt},2024}.
\end{aligned}$$

There are two important things to notice. First, nominal GDP and real GDP are the same in year 2020. This is simple because we decided to set 2020 as the base year, which also happens to be the current year in this case. Second, notice that starting in 2021, we use the prices that goods and services had in 2020 for real GDP calculations. Hence, if real GDP grows from year 2020 to 2024, it can only be because the amount of yogurt and bread produced in the economy has increased, since the prices are held fixed at their 2020 level.

2 Real Chain-Weighted GDP

While fixing prices at the level of some base year allows us to isolate output growth in (real) GDP calculations, it is also true that prices generally become outdated. For instance, computers today are much cheaper than they were twenty years ago, and education (e.g., college tuition) is much more expensive. Thus, fixing prices creates the problem that we may not weight goods and services in GDP calculations according to their current importance (see slides 33–34).

To overcome the weighting problems associated with constant prices, the BEA and other statistical agencies compute real GDP using a chained-weighted measure which is commonly referred to as "real chain-weighted GDP." The basic idea behind chain-weighting is that relative prices change over time, so that the base year should be updated year by year to reflect such changes and avoid the problems associated with real GDP at constant prices.

In contrast to real GDP at constant prices, real chained-weighted GDP uses information on prices from more than one year. For instance, prices from both year t and year $t+1$ are used to compute real growth from year t to $t+1$, prices from year $t+1$ and year $t+2$ are used to compute real growth from $t+1$ to $t+2$, and so on. Once we have obtained these year-to-year growth rates, we put them together to form a "chain" that is used to compute real GDP. In essence, the method for computing real chained-weighted GDP follows four steps:

1. **Compute the Laspeyres quantity index**:

$$(1+g_t)_{t-1} = \sum_i \underbrace{\frac{p_{it-1}q_{it-1}}{\sum_j p_{jt-1}q_{jt-1}}}_{\equiv \omega_{i,t-1}} \left(\frac{q_{it}}{q_{it-1}}\right) = \sum_i \omega_{it-1}\left(\frac{q_{it}}{q_{it-1}}\right).$$

   This index is named after the German economist *Ernst Laspeyres*, who introduced it in the 19th century. As you can see, we use prices from the earlier period (i.e., year $t-1$) to weight the growth of physical quantities (from year $t-1$ to year t). This index is a quantity index because we are ultimately interested in quantities. We weight the growth of quantities according to its relative importance in GDP.

   Example: Suppose there are only two goods in the economy, bread and yogurt. Then the Laspeyres quantity index from year 2020 to year 2021 is:

$$(1+g_{2021})_{2020} = \sum_{i=1}^{2} \underbrace{\frac{p_{i,2020}q_{i,2020}}{\sum_j p_{j,2020}q_{j,2020}}}_{\equiv \omega_{i,2020}} \left(\frac{q_{i,2021}}{q_{i,2020}}\right) = \sum_{i=1}^{2} \omega_{i,2020}\left(\frac{q_{i,2021}}{q_{i,2020}}\right).$$

If you expand the summation above, you will see that the Laspeyres quantity index weighs the growth in the quantity of bread produced from 2020 to 2021 using the share of bread sales in total sales in 2020, which reflects the relative importance of bread in the economy.

2. **Compute the Paasche quantity index**:

$$(1+g_t)_t = \sum_i \underbrace{\frac{p_{it} q_{it-1}}{\sum_j p_{jt} q_{jt-1}}}_{\equiv \omega_{i,t}} \left(\frac{q_{it}}{q_{it-1}}\right) = \sum_i \omega_{it} \left(\frac{q_{it}}{q_{it-1}}\right).$$

   This index is named after the German economist *Herbert Paasche*, who introduced it in the 19th century. In this case, we use prices from the later period (i.e., year t) to weight the growth of physical quantities (from year $t-1$ to year t).

3. **Compute Fisher's "ideal" chain index**:

$$1 + g_t = \sqrt{(1+g_t)_t \times (1+g_t)_{t-1}}.$$

   This index is named after the American economist *Irviing Fisher*, who introduced it in 1921. This index is the geometric mean of the Paasches and the Laspeyres quantity indices.

4. **Compute real chain-weighted GDP**:

$$\text{Real GDP}_t = \underbrace{(1+g_t)(1+g_{t-1})\cdots(1+g_{t-j})}_{\text{chained weight (of Fisher indices)}} \times \text{Nominal GDP}_{t-j-1}.$$

   Real chain-weighted GDP in year t, expressed in the units of some reference year $t-j-1$, is calculated by chaining all Fisher indices between year t and year $t-j$ and then multiplying that chain by the nominal GDP of the reference year.

   Example: Suppose we want to calculate real chain-weighted GDP in 2024 and express it in 2021 dollars. Then, we calculate:

$$\text{Real GDP}_{2024} = (1+g_{2024})(1+g_{2023})(1+g_{2022}) \times \text{Nominal GDP}_{2021}.$$

   Importantly, to compute real chain-weighted GDP in 2024, we use price information from the years 2021, 2022, 2023, and 2024. This method allows for constant adjustments of prices, overcoming the weighting problems associated with real GDP at constant prices. Recall that if we were to calculate real GDP using a fixed base year, say 2021, we would only use price information from that year.

3 Two Basic Results from Calculus Applied to Our Course

Basic arithmetic tells us that the percentage change of a product of two variables is approximately the sum of the percentage change in each variable, and that the percentage change of a ratio is approximately the percentage change in the numerator minus the percentage of the denominator. All we need to know to establish these two results is the product (or Leibniz) rule and the quotient rule from calculus.

3.1 Percentage change of the product of two variables

Suppose we are interested in understanding the behavior of nominal GDP over time. At any point in time, we can write nominal GDP in terms of the aggregate price P and the aggregate quantity Y; that is, nominal GDP is PY. To see how GDP behaves over time, we can take the total derivative of this product; that is, we can compute $\mathrm{d}(PY)$. Using the product rule, we have that[1]

$$\mathrm{d}(PY) = \mathrm{d}P \cdot Y + P \cdot \mathrm{d}Y.$$

We can divide both sides of this equation by nominal GDP, which yields

$$\frac{\mathrm{d}(PY)}{PY} = \frac{\mathrm{d}P \cdot Y}{PY} + \frac{P \cdot \mathrm{d}Y}{PY}.$$

Canceling common terms, we obtain

$$\frac{\mathrm{d}(PY)}{PY} = \frac{\mathrm{d}P}{P} + \frac{\mathrm{d}Y}{Y}.$$

This equation is written in "differential" notation, which describes an infinitesimal change in the product PY relative to PY. In practice, we do not observe infinitesimal changes in economic variables (e.g., GDP is not measured every other second), but rather larger changes that occur over longer periods of time; say, from year to year. For this reason, we often want to move from infinitesimal changes to finite changes. If $\mathrm{d}P$ and $\mathrm{d}Y$ are infinitesimal changes in price and quantities, we use ΔP and ΔY to denote finite changes. For instance, $\Delta P = P_t - P_{t-1}$ and $\Delta Y = Y_t - Y_{t-1}$. To move from differentials to finite differences, we approximate the infinitesimal changes by the

[1] In calculus, the product (or Leibniz) rule states that the derivative of the product two functions f and g is equal to the sum of two products. The first product is the derivative of the first variable multiplied by the second variable, and the second product is the first variable multiplied by the derivative of the second variable. Formally, $\mathrm{d}(f \cdot g) = \mathrm{d}f \cdot g + f \cdot \mathrm{d}g$. You may have also seen the product rule of calculus expressed as $(f \cdot g)' = f' \cdot g + f \cdot g'$. These two expressions are identical, only that the first one is written using Leibniz's notation, and the second one using Lagrange notation.

relative changes over finite intervals.[2]. In the case that concerns us, we write

$$\frac{\Delta PY}{PY} \approx \frac{\Delta P}{P} + \frac{\Delta Y}{Y}.$$

This equation states that the growth of nominal GDP is approximately equal to the sum of price growth and output growth.

3.2 Percentage change of the ratio of two variables

Now suppose we are interested in understanding the behavior of labor productivity (i.e., how many units of output are produced per hour worked). At any point in time, we can write labor productivity as Y/L, where Y represents units of the final good and L the total number of hours worked. To understand changes in labor productivity, we can take the total derivative of this ratio; that is, we can compute $\mathrm{d}(Y/L)$. Using the quotient rule, we have[3]

$$\mathrm{d}(Y/L) = \frac{\mathrm{d}Y \cdot L - Y \cdot \mathrm{d}L}{L^2}.$$

Canceling common terms and doing simple algebraic manipulations, we can write

$$\begin{aligned}\mathrm{d}(Y/L) &= \frac{\mathrm{d}Y}{Y}\frac{Y}{L} - \frac{Y}{L} \cdot \frac{\mathrm{d}L}{L} \\ &= \frac{Y}{L}\left(\frac{\mathrm{d}Y}{Y} - \frac{\mathrm{d}L}{L}\right).\end{aligned}$$

Rearranging,

$$\frac{\mathrm{d}(Y/L)}{Y/L} = \frac{\mathrm{d}Y}{Y} - \frac{\mathrm{d}L}{L}.$$

Approximating infinitesimal changes using finite intervals,

$$\frac{\Delta(Y/L)}{Y/L} \approx \frac{\Delta Y}{Y} - \frac{\Delta L}{L}.$$

This equation says that labor productivity growth is approximately equal to output growth minus hours growth.

[2] Such approximations are reasonable as long as changes are small enough.

[3] The quotient rule states that the derivative of the ratio of two functions f and g is given by

$$\mathrm{d}\left(\frac{f}{g}\right) = \frac{\mathrm{d}f \cdot g - f \cdot \mathrm{d}g}{g^2}.$$

You may have also seen the quotient rule stated as $(f/g)' = (f' \cdot g - f \cdot g')/g^2$. These two expressions are equivalent, only that one uses Leibniz's notation, and the other Lagrange notation.