## **Rules of Natural Logarithms** ECO 3302 – Intermediate Macroeconomics

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## **Rules of (Natural) Logarithms**

Natural logarithms are logarithms with base  $e \approx 2.7183$ . The natural logarithm of x is generally written as  $\ln(x)$ ,  $\log_e(x)$ , or  $\log(x)$  if the base e is implicit. The natural logarithm of x is the power to which e would have to be raised to equal x; that is,

$$e^{\ln x} = x$$

For example, the natural logarithm of 2,  $\ln 2$ , is 0.6931, because  $e^{0.6931} = 2$ . The basic rules for these logarithms are:

- Natural logarithm of 1 is 0 because  $e^0 = 1$ . Hence,  $\ln(1) = 0$ .
- Natural logarithm of *e* is 1 because  $e^1 = e$ . Hence,  $\ln(e) = 1$ .
- **Product rule**: The natural log of a product is the sum of the natural logs of the individual factors.

$$\ln(x \cdot y) = \ln x + \ln y.$$

• **Quotient rule**: The natural log a quotient is the difference of the natural logs of the numerator and denominator

$$\ln\left(\frac{x}{y}\right) = \ln x - \ln y.$$

• **Power rule**: The natural log of a number raised to an exponent is the exponent times the natural log of the base number.

$$\ln\left(x^{a}\right)=a\cdot\ln x.$$