ME2708 Economic Growth

Lecture 4: Introduction to Endogenous Growth

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Outline

Introduction I

- The main caveat of the Solow model is that technology, which could account for sustained growth, is exogenous
  - Inability to explain cross-country technology differences
- Good reasons to believe that technological progress however depends on economic factors (e.g. *profit-seeking firms and individuals*)
  - Move towards models of endogenous growth
- Endogenizing technology forces us to deal with IRS
  - If F exhibits CRS in K and L, Euler's theorem predicts that

$$F(K, L) = F_K(K, L)K + F_L(K, L)L$$

Or, more specifically,

$$Y(t) = R(t)K(t) + w(t)L(t)$$

i.e. the economy's output exhausted in paying factors of production, leaving no output to remunerate efforts in improving technology

Introduction II

- Two ways to solve this difficulty:
  1. Keep perfect competition but **introduce externalities**: technology as a *by-product* of some other economic activity, e.g. capital accumulation
  2. **Move away from perfect competition** and make technology dependent on intentional research efforts

- Simplest way to solve this issue is to think of technological progress as an externality (Arrow, 1962)
  - *Learning by doing*: unintended consequence of capital accumulation
  - Technological progress depends on aggregate capital, which *small, homogenous* firms are unable to influence!
    - $\Rightarrow$ firms take technology as given
  - Firms maximize profits by paying MPs to K and L, and their (*indirect*) contribution to technological progress is not remunerated

Introduction III

- The *learning-by-doing* externality introduced in AK-type models advances technological progress via capital accumulation
  - Externalities raise MPs, offsetting DRS, and give rise to production function $Y = AK$
- Why do we study AK models?
  - First generation of endogenous growth models
  - Important part of an economist toolkit
  - Shed some light on cross-country divergence
  - ...
- In particular we will study:
  - The Harrod-Domar model (Harrod, 1939; Domar, 1946)
  - Frankel's (1962) model
  - Lucas' (1988) model

The Harrod-Domar Model I

- Percursor of the AK model with functional form,

$$Y(t) = F[K(t), L(t)] = \min\{AK(t), BL(t)\}$$

  where A and B are fixed technological coefficients (*so there is no technological progress!*)

- Producing a unit of output requires $1/A$ units of capital and $1/B$ units of labor
  - No substitutability between inputs: if minimum requirements are not satisfied, there is no production!

- With fixed-technology coefficients, there is either capital surplus ($AK > BL$) or labor surplus ($AK < BL$)

The Harrod-Domar Model II

- Harrod-Domar focus on the second case in which capital is the limiting factor,

$$AK < BL \quad \Leftrightarrow \quad K < \left(\frac{1}{A}\right) BL$$

- Firms then produce according to,

$$Y(t) = AK(t) \tag{1}$$

hiring labor $(1/B)Y = (1/B)AK < L$

- With a fixed saving rate $s \in (0, 1)$, capital stock evolves according to,

$$\dot{K}(t) = sY(t) - \delta K(t) \tag{2}$$

The Harrod-Domar Model III

- Substituting (1) into (2),

$$\dot{K}(t) = sAK(t) - \delta K(t)$$

- The growth rate of capital is then,

$$g_K = \frac{\dot{K}(t)}{K(t)} = sA - \delta$$

- Because Y is *strictly* proportional to capital,

$$g = g_Y = g_K$$

- The growth rate g is increasing in the saving rate s

The Harrod-Domar Model IV

- The main criticism to the **Harrod-Domar Model** is its **inability to account for sustained growth in per capita terms** (*see below*!)

- If we introduce population growth n, we can express the capital accumulation equation in per capita as,

$$\frac{\dot{\tilde{k}}(t)}{\tilde{k}(t)} = \frac{\dot{K}(t)}{K(t)} - n = \frac{sY(t) - \delta K(t)}{K(t)} - n = sA - \delta - n$$

- The growth rate in per capita terms is then,

$$g = g_{\tilde{y}} = g_{\tilde{k}} = sA - \delta - n$$

- If $g = sA - \delta - n > 0$, the capital stock per capita increases up to a point where K is no longer the limiting factor, but instead L is!
  - Then the production function becomes $Y(t) = BL(t)$
  - which implies that Y grows at rate n and output per capita, $\tilde{y}$, therefore ceases to grow!

Frankel's Model I

- Frankel's (1962) is the first AK model that can account for sustained growth in *per capita* output
- This model aims to combine the virtues of both the basic Solow and the Harrod-Domar model
  - **From Solow**: perfect competition, substitutable inputs, full empl.
  - **From Harrod-Domar**: long-run growth rate depends on savings rate s
- Frankel builds in Arrow (1962) and introduces the externality of learning by doing through capital accumulation
- The economy is populated by firms $i = \{1, 2, \dots, N\}$ that produce according to,

$$y_i(t) = \overline{A} k_i(t)^{\alpha} l_i(t)^{1-\alpha}$$

where k_i and l_i are the firm's specific capital and labor, and $\overline{A}$ is aggregate productivity

Frankel's Model II

- Aggregate productivity depends on total capital,

$$\overline{A} = A(0)\left(\sum_{i=1}^{N} k_i(t)\right)^{\eta}$$

  where η is a positive coefficient capturing the extent of the externality

- We normalize labor (e.g. $l_i = 1, \forall i$) for simplicity

- Aggregate capital and output can then be denoted as,

$$K(t) = \sum_{i=1}^{N} k_i(t) \quad \text{and} \quad Y(t) = \sum_{i=1}^{N} y_i(t)$$

- Firms have access to the same technology and face equal factors prices so that factors of production are allocated uniformly,

$$k_i(t) = \frac{K(t)}{N}, \quad \forall i$$

Frankel's Model III

- Aggregate technology can then be re-expressed as,

$$\overline{A} = A(0)K(t)^{\eta}$$

- Thus, firms' output is equal to

$$\begin{aligned} y_i(t) &= \overline{A}k_i(t)^{\alpha} \\ &= A(0)K(t)^{\eta}\left(\frac{K(t)}{N}\right)^{\alpha} \end{aligned}$$

- And aggregate output is,

$$\begin{aligned} Y(t) = \sum_{i=1}^{N} y_i(t) &= N\left(A(0)K(t)^{\eta}\left(\frac{K(t)}{N}\right)^{\alpha}\right) \\ &= A(0)N^{1-\alpha}K(t)^{\eta+\alpha} \end{aligned}$$

- If we define $A = A(0)N^{1-\alpha}$, we can express Y as,

$$Y(t) = AK(t)^{\eta+\alpha}$$

Frankel's Model IV

- As in Solow, we have constant saving rate s so that the capital accumulation equation is,

$$\begin{aligned}\dot{K}(t) &= sY(t) - \delta K(t) \\ &= sAK(t)^{\eta+\alpha} - \delta K(t)\end{aligned}$$

- The growth rate is then,

$$g_K(t) = \frac{\dot{K}(t)}{K(t)} = sAK(t)^{\eta+\alpha-1} - \delta \tag{3}$$

- We can differentiate between three cases now:
  1. $\eta + \alpha < 1$: knowledge spillovers η do not compensate for DRS in K
  2. $\eta + \alpha > 1$: knowledge spillovers η counteract DRS in K
  3. $\eta + \alpha = 1$: knowledge spillovers η exactly compensate DRS in K

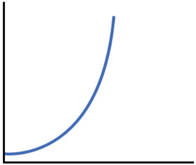
Frankel's Model V

1. $\underline{\eta + \alpha < 1}$: knowledge spillovers η do not compensate for DRS in K
   - Steady-state capital stock is,
   $$K^* = \left(\frac{sA}{\delta}\right)^{\frac{1}{1-\eta-\alpha}}$$
   - Eq. (3) can be rewritten as,
   $$g_K(t) = \frac{\dot{K}(t)}{K(t)} = sA\left(\frac{1}{K(t)^{1-\eta-\alpha}}\right) - \delta$$
   - Long-rung growth rate is zero
   - Same dynamics as in basic Solow with no technological progress and no population growth:
     - $K > K^*$: negative capital growth until $K = K^*$ (g_K is a decreasing function of K)
     - To do now: calculate K^* for an economy with the following parameter values $s = 0.2, A = 5, \delta = 0.05, \eta + \alpha = 0.4$. Calculate g_K for $K_1 = 50, K_2 = 147.3613$ and $K_3 = 300$ (*you have 5 min!*)

Frankel's Model VI

2. $\underline{\eta + \alpha > 1}$: knowledge spillovers η counteract DRS in K

   - Steady-state capital stock is not an stable equilibrium (g_K is now an increasing function of K)
   - Dynamics: Even if $K > K^*$, K keeps on rising at an ever-increasing growth rate $\Rightarrow$ explosive growth case!



- To do now: calculate K^* for an economy with the following parameter values $s = 0.2, A = 5, \delta = 0.05, \eta + \alpha = 1.1$. Calculate g_K for $K = 50$ (*you have 3 min!*)

Frankel's Model VII

3. $\underline{\eta + \alpha = 1}$: knowledge spillovers η exactly compensate DRS in K
   - We get the production function $Y = AK$
   - Output is proportional, but not identical, to K
   - $g_K = sA - \delta$ as in Harrod-Domar, but now with substitutable factors and market clearing
   - Output per person grows at $g_{\tilde{y}} = sA - \delta - n$
   - The economy always grows at constant rate $sA - \delta - n$, regardless of the initial level of capital-labor ratio!
   - The convergence result vanishes!
   - **Unlike in Harrod-Domar model**, permanent increases in s raise the growth rate permanently

Important observation

Recall from Lectures 2&3 that $Y = AK$ could also be obtained from nesting the Cobb-Douglas production function by setting $\alpha = 1$ and violating neoclassical technology assumptions $KA1$ and $KA2$.

Lucas' Model I

- Lucas' (1988) growth model attributes a central role to human capital in the aggregate production function, which takes the form,

$$Y(t) = F[K(t), h(t)L(t)] = K(t)^{\alpha}(h(t)L(t))^{1-\alpha}$$

  where h denotes human capital per capita

- Lucas' model resembles the basic Solow model with labor-augmenting technological change
  - Only that now we have labor-augmenting human capital!

- Human capital evolves according to,

$$\dot{h}(t) = (1-u)h(t)$$

  where u is the time spent working, and $(1-u)$ is the time spent accumulating skills

- The growth rate of human capital is then,

$$\frac{\dot{h}(t)}{h(t)} = 1-u$$

Lucas' Model II

- We can express the production function in per capita terms, i.e. $\tilde{y}(t) = Y(t)/L(t)$, as

$$\tilde{y}(t) = \tilde{k}(t)^{\alpha} h(t)^{1-\alpha}$$

- Taking logs and time derivatives capital grows according to,

$$\begin{aligned}
\frac{\dot{\tilde{k}}(t)}{\tilde{k}(t)} &= \frac{\dot{K}(t)}{K(t)} - \frac{\dot{L}(t)}{L(t)} \\
&= \frac{sY(t)}{K(t)} - (\delta + n) \\
&= \frac{sY(t)}{\tilde{k}(t)L(t)} - (\delta + n) \\
&= \frac{s\tilde{y}(t)}{\tilde{k}(t)} - (\delta + n)
\end{aligned}$$

Lucas' Model III

- Output per capita growth rate in this economy is given by,

$$g_{\tilde{y}} = g_{\tilde{k}} + (1 - u)$$

- Output per capita growth is results from both growth of the capital stock per capita and also from increases in relative skills

- Governments' ability to permanently increase $(1 - u)$ (time spent in accumulating skills) can permanently increase growth of output per worker!

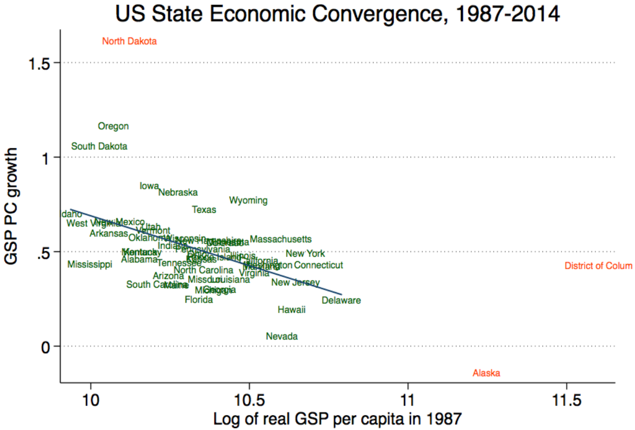
Criticisms I

- AK-type models violate neoclassical technology assumptions, requiring the production function to be linear in the capital stock
  - Imposing $\alpha = 1$ implies that national income is *only an fully* used to remunerate capital (*contradicts empirical evidence*)
  - No diminishing returns to capital: $F_{KK} \not< 0$
  - Violation of Inada conditions:

$$\neg \lim_{K \to \infty} F_K = 0 \quad \textit{but rather} \quad \lim_{K \to \infty} F_K = A > 0$$

- Although growth is endogenous:
  - It is not sustainable (Harrod-Domar model, Lucas model)
  - Relies on *external* accumulation of knowledge (Frankel's model)
  - Relies on human capital (Lucas model)

- Even when growth was sustainable (Frankel's model with $\eta + \alpha = 1$), the model cannot explain convergence. . . but rather *divergence*!
  - Useful to explain poverty traps and widening gaps between developed- and developing- countries!

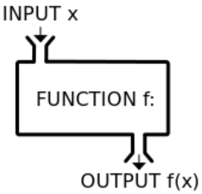


Criticisms III

- Although the previous models are endogenous. . .
  - . . . technological progress is still no convincingly accounted for!
- Empirical inquiries in this and previous courses (*ME2720*) emphasized that in a satisfactory model technology should enjoy a prominent role
- So we move to *endogenous, innovation-based* models of technological change which:
  - allow technological progress to respond to economic incentives
  - allow to introduce richer market structures and more-detailed prescriptions for government intervention
- . . . but let's first introduce some valuable theoretical concepts around which these models revolve!

Technology I

- In growth models, technology is directly and indirectly linked to production, i.e. it is part of the "machine" that converts inputs into output



- Recall that in general production functions of the type,

$$Y = F[A(t), K(t), L(t)]$$

technology is usually reflected by the term $A(t)$, attending to standard notation!

Technology II

- In the Cobb-Douglas production function with labor-augmenting technology,

$$Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$$

  $A(t)$ is thought of as an index of technology at time t

- Importantly, α is also part of the technology as it influences its behavior!

- With one *homogeneous* final good, better technology means that more (*or equal*) output can be produced with same (*or less*) inputs

- Technological improvements result usually from innovations, usually distinguished:
  1. **By type**: process- (e.g. business and organizational models) vs. product- (e.g. physical goods) innovations
  2. **By impact**: radical- (or macro) vs. incremental- (or micro) innovations

Technology III

Innovations I

- Innovations by type:
  1. **Process innovations**: *higher-quality products* (e.g. ICA offers fresher fish by halving transport time) and/or *cheaper products* (e.g. ICA's better internal organization allows reducing consumer prices by 5%)
  2. **Product innovations** relate to the improvement of existing goods (e.g. higher-resolution TV) or the introduction of new goods (e.g. invention of the smartphone)

- Innovations by impact:
  1. **Radical innovations**: alter production processes in general (e.g. printing press, electricity, steam engine, telephone, ICE, airplane, computers, internet, etc.)
  2. **Incremental innovations**: improve existing products (energy-saving vehicles, longer battery-life smartphones, more efficient programming languages, etc.)

- Heterogenous agents with *different preferences* for quantity, quality, and type of products provide a clear intuition for the emergence of multiple and different innovations
- Most endogenous models do not incorporate different types of innovations. . .
- . . . rather, innovation-based models of endogenous technological change roughly fall in two categories:
  1. **Product-variety models**: new, but not necessarily improved, products (following the pioneering contribution of Romer (1990))
  2. **Schumpeterian growth theory**: incremental innovations arise as as result of creative destruction (following the pioneering contribution of Aghion and Howitt (1992))

Technology V

- In models of endogenous technology, firms and/or individuals must be able to choose among types of technologies or, at least, be able to influence the invention of new technologies
- A plausible assumption is then that economic agents can influence the probability of discovering new technologies by:
  - putting greater effort, i.e. *more time*
  - investing more money, i.e. *higher R&D funds*
- This reasoning often translates into the introduction of the research sector or the R&D production function
  - In modeling the "unknown", the technology production functions needs not be deterministic
  - Outputs may not be known beforehand but, on average, influenciable

Technology and Ideas I

- Arguably, technological innovations materialize through ideas

- Romer (1986) formalized the relationship between economics of ideas and economic growth

Ideas $\longrightarrow$ Nonrivalry $\longrightarrow$ Increasing Returns $\longrightarrow$ Imperfect Competition

- Ideas are **non-rivalrous**: the use of an idea by one producer to increase efficiency does not preclude its use by others
  - This contrasts with the use of capital and labor (and most goods in general, e.g. subway spot, apartment in Stockholm, etc.)!

Technology and Ideas II

- Although ideas are *non-rivalrous*...specialized labor, which is rivalrous, is required to understand the blueprints embedded in a technical document
- Nonrivalry of ideas implies:
  - **Increasing returns to scale** (IRS): advancements in A readily available to all producers, either at time t or at $t+1$
  - **Market size effects**: the profitability of investing in one's idea may depend on its potential market

"*It is not worth my while to manufacture your engine for three countries only, but I find it very well worth my while to make it for all the world*" $\sim$ M. Boulton to James Watt

- To produce a technological innovation has a fixed cost but once a new invention is created, production of additional units is relatively cheap!

Technology and Ideas III

- Ideas are **partially excludable**: the patentee can be granted temporary monopoly rights thanks to IPRs and patenting laws
  - Notable exceptions: computer software, business models, mathematical methods, economic theories, etc.

- Goods can be categorized attending to their degree of rivalry and excludability

	Excludable	**Non-excludable**
Rival	PRIVATE GOODS Clothing, food Accommodation Personal devices	COMMONS GOODS Forests, pastures Water, atmosphere Wild animals
Non-rival	TOLL GOODS Netflix SATS Economic Journals	PUBLIC GOODS National defence Street lights Basic R&D

Technology and Ideas IV

- Excludable goods allow producers to appropriate their benefits

- Non-excludable goods do not allow producers to *fully* appropriate their benefits and give rise to externalities
  - Goods with **positive externalities** (basic R&D, energy-saving goods) are underproduced
  - Goods with **negative externalities** (polluting goods) are overproduced

Technology and Ideas V

- Rivalrous goods must be produced each time they are sold (e.g. n people demanding iPhone, n iPhones must be produced)
- Non-rivalrous goods must be produced only once (e.g. iPhone apps, synthetic formulas for vaccines, etc.)
  - Ideas themselves have a fixed cost of production but zero marginal cost $\Rightarrow$ IRS
  - Most innovations: fixed cost and small marginal cost, e.g. vaccines, mobile apps

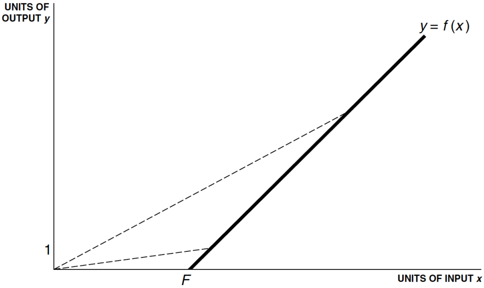
Technology and Ideas VI

- Production of innovative products could be proxied by

$$y = f(x) = 100(x - F)$$

where F is the fixed cost and x is the marginal cost

- *Book example*: to produce first pill y the firm needs to invest F research (skilled) hours; then each additional (unskilled) labor hour x produces additional 100 pills



Technology and Ideas VII

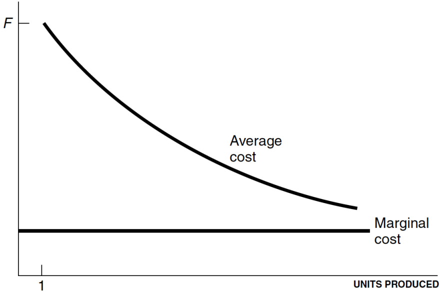
- Production function $y = f(x) = 100(x - F)$ exhibits IRS, i.e.,

$$f(\lambda x) > \lambda(fx), \quad \forall \lambda > 1$$

Doubling inputs more than doubles output

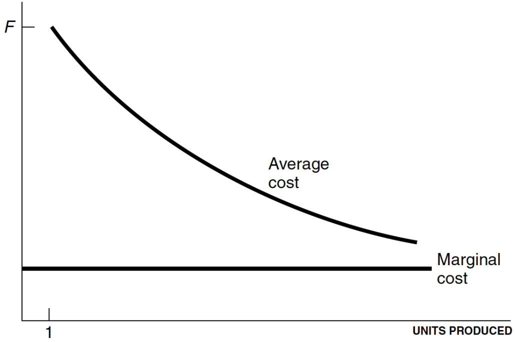
$$y = f(x) = 100(x - F) \begin{cases} x = 10, F = 1 & y = 900 \\ x = 20, F = 1 & y = 1900 \end{cases}$$

- IRS can also be seen by looking at average costs



Technology and Ideas VIII

- The fact that ACs > MC (*asymptotically* $AC = MC$) gives a motivation for moving away from perfect competition
  - If $AC > MC$, setting $P = MC$ implies negative profits
  - No incentive for firms to leverage F and develop product y
  - To develop new products firms must be able to set $P > MC$, introducing the possibility of earning positive profits



Ideas, Innovation and Profits I

- Are technological innovations profit-driven?
  - **No**, science-driven: autonomous progress, talented individuals, little emphasis on profit opportunities
  - **Yes**, profit-driven (Griliches and Schmookler, 1963)

    "*The labor of Watt (...) was undergone in the prospect of a remuneration from the producers*" – J.S. Mill

    "*An engineer's life without patent was not worthwhile*" – James Watt

    "*Invention is largely an economic activity which, like other economic activities, is pursued for gain*" – Schmookler

- Schmookler's (1966) analysis of several industries concludes that the stimulus for innovation was largely "*a potentially profitable opportunity to be seized*"

Ideas, Innovation and Profits II

- If innovation responds to profit incentives, *market size* must play a crucial role
  - "The amount of invention is governed by the extent of the market" (Schmookler, 1966)
  - Example: recurrent horseshoes' innovations between mid-*XIX* and mid-*XX* century, only stopped when ICE displaced the horse

- Papers documenting that innovation is profit-driven:
  - Newell, Jaffe and Stavins (1999): air-conditioners more energy-efficient between 1980-1990 as a response to higher energy prices
  - Popp (2002): patents for energy-saving technologies positively affected by energy prices
  - Finkelstein (2004) and Acemoglu and Linn (2004): pharmaceuticals develop more "relevant" vaccines (i.e. potentially higher demand)
  - Aghion *et al.* (2016): patents in clean vehicles increase as a response to higher energy prices whilst patents in dirty vehicles decrease
  - Lööf and Perez (2017): solar industry's development encouraged by higher energy energy prices

Ideas, Innovation and Profits III

- Recall that innovations require large, one-time costs and in order to be profitable producers must be able to appropriate profits
- Without IPRs, most innovations would not be profitable (exceptions: Coke's formula, Google's algorithm, etc.)
  - Reverse engineering (e.g. chemical analyses of drugs)
- Some economists even claim that establishment of IPRs is responsible for modern, sustained growth

"*Throuhout man's past he has continually developed new techniques, but the pace has been slow and intermittent. The primary reason has been that incentives for developing new techniques have occurred only sporadically. Typically, innovations could be copied at not cost by others and without any reward to the inventor or innovator. The failure to develop systematic property rights in innovation up until fairly modern times was a major source of the slow pace of technological change*" – D. North

Population and Ideas I

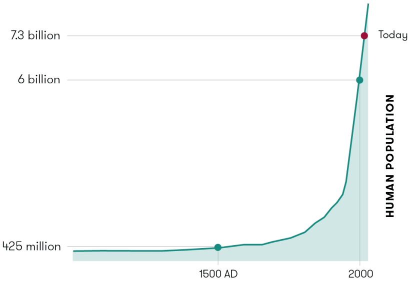
- Total number of ideas partly and positively depends on population size
- Given that ideas are non-rivalrous and have potential to increase productivity… *the more, the merrier*!

  "*One can hardly imagine how poor we would be today were it not for the rapid population growth of the past (…) If I could re-do the history of the world, halving population size each year from the beginning of time on some random basis, I would not do it for fear of losing Mozart in the process*"
  – E. Phelps

- The idea that greater population may serve as an opportunity for increased growth is counterintuitive to many economic theories (e.g. *Malthusian* economics, Solow model)
  - which build on *rivalrous* goods $\Rightarrow \uparrow$ population, $\downarrow$ goods per capita
- In models of endogenous technological change, however, population is often the engine of growth!

Population and Ideas II

- Positive association between population and ideas
  - Reasonably good proxies
  - ...shortcomings: people's productivity, patentability issues (e.g. software, patents with different economic value, etc.)



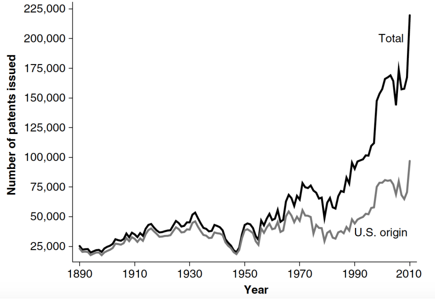


Figure: World population size (*left*) vs. patents issued in the US (*right*)

Take Outs I

- Two ways to endogenize technology:
  1. Introduction of **externalities**, e.g. *learning by doing*
     $\Rightarrow$ AK models (Harrod-Domar, Frankel's, Lucas', etc.)
  2. Make **technology dependent on intentional research efforts**
     $\Rightarrow$ Innovation-based models (Romer, Jones, Schumpeterian...)
- The **first-generation models of endogenous growth**, despite of its simplicity, **not satisfactory**:
  - **Violate neoclassical technology assumptions**
  - **Growth is not sustainable** (Harrod-Domar's and Lucas models)
  - ...and when it is (Frankel's model special case with $\eta + \alpha = 1$), it is based on external accumulation of knowledge rather than in intentional technological progress!
  - **Rule out the convergence result**
  - **contradicting empirical evidence**

Take Outs II

- Innovations often distinguished by **type** (process vs. product) and/or **impact** (incremental vs. radical)
- Innovation-based growth models fall roughly in 2 categories: **product-variety models** vs. **Schumpeterian models**
  - Firms able to influence technology, e.g. higher R&D investment
  - Introduction of R&D production function
- Innovations result from ideas, which are **non-rivalrous and partially excludable**
  - *Increasing returns to scale*, evident when looking at average costs
  - *Market size effects*
- Innovation is **profit-driven**. For innovation to be profitable:
  - innovators able to appropriate (*at least temporarily*) profits ⇒ **IPRs**
  - **prices set above marginal costs** ⇒ no perfect competition
- **Endogenous technological change models**, in contrast to previous economic theories, point to **population as growth engine**!

Further Reading

1. Chapter 9 & 4 in Jones' (1990) textbook
2. Chapter 2 (*up to section 2.5*) in Aghion and Howitt's (2009) book

Thank you for your attention!