ME2708 Economic Growth

Assignment 2
Endogenous Growth Models

Deadline: April 23, 2018 at 13:15[†]

Instructions

You must solve the first 4 exercises, the rest is left as preparation material for the exam. Grading is according to the **P-F** scale, but demands are high. You are allowed to work in groups of (*maximum*) three persons; individual work is not only acceptable but also encouraged. When you are asked to derive something, you must show *mathematically* how you came with the given answer[1]. Presentations in exercise classes are voluntary.

Something you might appreciate to know is that, according to previous experience, students who do not intensively work on the assignments struggle to pass this course. The nice thing, however, is that this practice handsomely pays off in the written examination.

[†]Submission of solutions may be done either in person at the beginning of the class or by email to luis.perez@indek.kth.se.

[1]Providing only final answers implies failure of the assignment.

1 The AK Model

Suppose the economy contains many profit-maximizing firms, indexed by i, that produce homogenous good Y. Each firm uses a fracion of labor L and a fraction of capital K. The production of individual firms is then given by

$$Y_i(t) = K_i^{\alpha}(t)[B(t)L_i(t)]^{1-\alpha}, \quad 0 < \alpha < 1$$

where K_i and L_i denote the capital and labor used by firm i, respectively. B captures the general level of technology, which is exogenous and available to all firms. The aggregate production function can then be expressed as,

$$Y(t) = \sum_{i=1}^{I} Y_i(t) = K^{\alpha}[B(t)L(t)]^{1-\alpha}, \quad 0 < \alpha < 1$$

(a) Assume that the general level of technology depends positively on the aggregate level of the capital stock K and some positive constant A so that $B(t) = A^{\eta}K(t)^{\zeta}$. What parameter restrictions shall be imposed on η and ζ in order for the production function to be equal to $Y(t) = AK(t)$ assuming that labor L is constant and equal to 1?

2 AK vs. Basic Solow's Model I

Derive the per capita production function and capital accumulation equation for the Harrod-Domar model[2], and take the basic Solow model with technological progress and population growth as a benchmark.

(i) Contrast their main results with respect to long-run growth and convergence.

(ii) Based on the evidence presented so far in this course, which framework do you believe to receive more support from the data? Reason why.

[2]Note that capital is initially the limiting factor.

3 Romer's Model

Consider two economies, i and j, in Romer's setting, where the following set of equations hold:

$$Y(t) = F[K(t), A(t)L_Y] = K(t)^\alpha [A(t)L_Y]^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{1}$$

$$\dot{K}(t) = s_K Y(t) - \delta K(t), \qquad 0 < s_k, \delta < 1 \tag{2}$$

$$\dot{A}(t) = \theta L_A A(t), \qquad \theta > 0 \tag{3}$$

$$L = L_Y + L_A, \qquad L_Y, L_A > 0 \quad \forall t \tag{4}$$

$$s_R = L_A / L \tag{5}$$

This exercise asks you to:

(i) Transform the system of equation in order to obtain a state variable that is constant along a balanced growth path (*BGP*). In particular, transform the aggregate production function and the aggregate capital accumulation equation such that you obtain their counterparts in terms of effective units of labor. Show your calculations.

(ii) Derive the steady-state capital- and output- in terms of effective units of labor.

(iii) Obtain the steady-state output per capita equation and interpret it!

(iv) Calculate output, consumption, and investment (*all in per capita terms*) for both country i and country j in year $t = 4$. Both countries have identical parameters. More specifically,

$$\epsilon_i = \epsilon_j = (\alpha, s, \delta, g_A, \theta, s_R) = (1/3, 0.28, 0.05, 0.02, 0.07, 0.1)$$

Initial levels of technology do, however, differ:

$$A_i(0) = 20, \qquad A_j(0) = 15$$

Finally note that $L = 200$.

(iv) Maximize output per capita with respect to the share of researchers (*show your calculations*!) and give optimal values for the research sector in these two countries.

4 Schumpeterian Growth

Start by highlighting the main differences between product-variety and Schumpeterian growth models. Then consider the following setting:

$$Y(t) = F[K(t), A_i L_Y(t)] = K(t)^\alpha [A_i L_Y(t)]^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{6}$$

$$\dot{K}(t) = s_K Y(t) - \delta K(t), \qquad 0 < s_k, \delta < 1 \tag{7}$$

$$\mathbb{E}\left[\frac{\dot{A}_i}{A_i}\right] = \gamma\overline{\mu} L_A(t) = \gamma\theta \frac{L_A(t)^\lambda}{A_i^{1-\phi}}, \qquad 0 < \lambda, \phi < 1, \;\; \theta > 0, \gamma > 1 \tag{8}$$

$$\dot{L}(t) = nL(t), \qquad n > 0 \tag{9}$$

$$L(t) = L_Y(t) + L_A(t), \qquad L_Y, L_A > 0 \quad \forall t \tag{10}$$

$$s_R(t) = L_A(t)/L(t) \tag{11}$$

Answer the following questions:

(i) Derive the aggregate and the per capita growth rates of this economy along a *BGP*.

(ii) What structural characteristics you expect to observe in growth-leading economies?

5 Neoclassical Assumptions and Dynamics

Consider the following production function:

$$Y(t) = AK(t) + BK(t)^\alpha L(t)^{1-\alpha}, \qquad 0 < \alpha < 1, \quad A, B > 0$$

(i) Does this production function satisfy neoclassical technology assumptions?

(ii) Assume depreciation at rate $\delta \in (0, 1)$ and population growth at rate $n > 0$. Show, by growing the growth rate of per capita capital stock as a function of per capital capital stock, that this specification allows for transitional dynamics.

(iii) Assume $sA > n + \delta$. What is the growth rate of per capita capital when $\tilde{k} \to \infty$.

6 AK vs. Basic Solow's Model II

Derive the per capita production function and capital accumulation equation for Frankel's model[3], and take the basic Solow model with technological progress and population growth as a benchmark.

(i) Contrast their main results with respect to long-run growth and convergence.

(ii) Based on the evidence presented so far in this course, which framework do you believe to receive more support from the data? Reason why.

Further Exam Prep.

You can find additional exercises in Chapters 4&5 in the main (Jones') textbook of this course.

[3]Recall that in Frankel's model there are three possibilities. Make sure to take all of them into account.