ME2708 Economic Growth

Assignment 1
Exogenous Growth Models

Deadline: March 29, 2017 at 13:15[†]

Instructions

You must solve the first 4 exercises, the rest is left as preparation material for the exam. Grading is according to the **P-F** scale, but demands are high. You are allowed to work in groups of (*maximum*) three persons; individual work is not only acceptable but also encouraged. When you are asked to derive something, you must show *mathematically* how you came with the given answer[1]. Presentations in exercise classes are voluntary.

Something you might appreciate to know is that, according to previous experience, students who do not intensively work on the assignments struggle to pass this course. The nice thing, however, is that this practice handsomely pays off in the written examination.

1 Neoclassical Production Functions

Functions that satisfy key assumptions *KA1* and *KA2* are categorized as neoclassical production functions. Show that the Cobb-Douglas production function $Y = K^{\alpha}L^{1-\alpha}$ where $0 < \alpha < 1$ and $K, L > 0$ satisfies these assumptions. Namely, show that: i) it exhibits constant returns to scale in capital and labor; ii) it is characterized by positive and diminishing returns to capital and labor; and, finally, iii) it satisfies the Inada (1962) conditions.

[†] Submission of solutions may be done either in person at the beginning of the class or by email to luis.perez@indek.kth.se.

[1] Providing only final answers implies failure of the assignment.

2 The basic Solow model

Suppose the economy is described by the following set of equations:

$$Y(t) = F[A(t), K(t), L(t)] = K(t)^{\alpha} L(t)^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{1}$$

$$\dot{K}(t) = sY(t) - \delta K(t), \qquad 0 < \delta < 1 \tag{2}$$

$$\dot{L}(t) = nL(t) > 0 \tag{3}$$

where equations (1), (2) and (3) represent the production function, the motion of (physical) capital accumulation and the behavior of population growth, respectively. This exercise asks you to:

(a) Express the production function and the capital accumulation equation in per capita terms.

(b) Derive the steady-state capital stock per capita ($\tilde{k}^*$) and the steady-state output per capita ($\tilde{y}^*$). What are the growth rates for total capital stock K and total output Y?

(c) Derive the savings rate that maximizes consumption per capita in the steady state. What is the marginal product of capital in this steady state?

(d) Illustrate the steady state of the economy in a figure where consumption per capita is maximized. Denote $\tilde{y}^*, \tilde{k}^*, \tilde{c}^*$ as well as depict the production function, the capital accumulation equation and the gross investment function.

3 Shocks in the basic Solow model

Assume that we are in a fictitious economy that, at the time of its 45^{th} presidential elections, follows a balanced growth path (BGP). The newly elected president has the intention to present his somehow controversial policies to both Congress and Senate to ask for approval.

(a) The current population growth rate of the economy is 1%, a number which is entirely driven by migration flows (the domestic population is stagnated). Since the new elected president considers immigration a threat to both national defence and domestic thriveness, he is determined to not only ban the entry of new migrants but also to deport a fraction of former migrants. The result of these policies translate into a negative population growth rate of −1%. What happens to the BGP values of capital per worker, output per worker, and consumption per worker? Sketch the paths of these variables as the economy moves to its new BGP.

(b) Despite immigration policies were implemented, the recently-elected government is not completely satisfied. It considers that the previous government's investments in public goods and services were to a great extent unjustified (proper of a "*paternalistic*" state). As such, it decides to cut down the public investment's rate, which translates in a reduction of the total investments rate. Show graphically how the new policy, both separately and in addition to the former immigration policies, could affect the economy and its BGP.

(c) Describe the effect of the fall in both population growth and investments rate on the path of output (that is, total ouput, not output per worker).

4 Basic Solow with technological progress

Suppose the economy is described by the following set of equations:

$$Y(t) = F[K(t), A(t)L(t)] = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{4}$$

$$\dot{K}(t) = sY(t) - \delta K(t), \qquad 0 < \delta < 1 \tag{5}$$

$$\dot{L}(t) = nL(t) > 0 \tag{6}$$

$$\dot{A}(t) = g_A A(t) > 0 \tag{7}$$

where equations (4), (5), (6) and (7) represent the production function, the motion of (physical) capital accumulation, the behavior of population growth and the development of technology, respectively. This exercise asks you to:

(a) Express the production function and the capital accumulation equation per effective units of labor.

(b) Derive the steady-state capital stock ($\widehat{k}^*$) and the steady-state output ($\widehat{y}^*$) per effective units of labor.

(c) Calculate steady-state equilibrium quantities for capital, output, consumption and investment in (b) when $(\alpha, s, \delta, n, g_A) = (1/3, 0.2, 0.05, 0.01, 0.03)$.

5 Convergence in the basic Solow Model

The speed of convergence in the basic Solow model with technological progress is equal to $(1-\alpha)(\delta + g_A + n)$. Assume that a given Swedish region, say Värmland, presents the following parameter values:

$$(\alpha, \delta, g_A, n) = (1/4, 0.05, 0.01, 0.01)$$

One then wonders how many years will it take for Värmland to reach halfway towards its steady state?

6 Augmented Solow model

Verify that derivation of Equation (36) in lecture slides 2 & 3 is correct. Namely, show that the steady-state equilibrium values in the augmented Solow model conform according to:

$$\widehat{k}^* = \left(\left(\frac{s_k}{\delta_k + g_A + n} \right)^{1-\beta} \left(\frac{s_h}{\delta_h + g_A + n} \right)^{\beta} \right)^{\frac{1}{1-\alpha-\beta}}$$

$$\widehat{h}^* = \left(\left(\frac{s_k}{\delta_k + g_A + n} \right)^{\alpha} \left(\frac{s_h}{\delta_h + g_A + n} \right)^{1-\alpha} \right)^{\frac{1}{1-\alpha-\beta}} \tag{8}$$

Further Exam Prep.

You can find additional exercises in Chapters 2&3 in the main (Jones') textbook of this course.